Nano tech Hmw

Chap 2 (1, 3, 7, 10).

2.1: energy of photon: $E = h\nu = \frac{hc}{\lambda} = \frac{1240\ eV\cdot nm}{650\ nm} = 1.91\ eV$ or $3.06E{-}19\ J$

energy of electron: (non-relativistic)

$$E_k = \frac{p^2}{2m_e} = \frac{h^2}{2m_e\lambda^2} = \frac{(hc)^2}{2m_ec^2\cdot\lambda^2} = \left(\frac{1240\ eV\cdot nm}{650\ nm}\right)^2 \cdot \frac{1}{2\times 0.511\ MeV}$$

$= 3.56E{-}6\ eV$ or $5.705\ E{-}25\ J$

(relativistic: $E_k = \sqrt{p^2c^2 + m_e^2c^4} - m_ec^2$ ~~p...m~~

$p\cdot c = \frac{hc}{\lambda} = E_{photo} = 1.91\ eV << m_e c^2 = 0.511\ MeV$

→ yield same answer as non-relativistic)

2.3: $E = h\nu = 6.63\times10^{-34} \times \begin{cases} 60\ Hz & \simeq 3.976\times10^{-32} J = 2.481\times10^{-13} eV \\ 2.4\times10^{9}\ Hz & = 1.59\times10^{-24} J = 9.926\times10^{-6} eV \\ 3\times10^{16}\ Hz & = 1.988\times10^{-17} J = 124.07\ eV \end{cases}$

2.7: wave length of baseball:

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.626\times10^{-34}}{0.15\ kg\cdot 90\ m/h} = 1.767\times10^{-31}\ (m).$$

The scale of baseball bats is much larger than wave length

→ diffraction not happen.

2.10: electron, $\lambda = \frac{h}{\sqrt{2m_eE}} = \frac{1240.7}{\sqrt{2\times4\times0.511\times10^6}}\ (nm) = 6.14\times10^{-10}\ m$ ~~$6.14\times10^{-19}\ m$~~

$E = 4eV$

proton: $\lambda = \frac{h}{\sqrt{2m_pE}} =$ ~~...~~ $1.431\times10^{-11}\ m$

photon: $\lambda = \frac{hc}{E} = \frac{1240.7\ eV\cdot nm}{4\ eV} = 310.175\ nm = 3.10175\times10^{-7}\ m$

all three particles have same frequence $f = \frac{E}{h} =$ ~~9.672×10^{-14}~~ (Hz)

momentum: electron: $p_e = \sqrt{2m_eE} = 1.08\times10^{-24}\ (kg\ m\cdot s^{-1})$.

proton: $P_{proton} = \sqrt{2m_pE} = 4.63\times10^{-23}\ (kg\ m\ s^{-1})$

photon: $P_{photon} = \frac{E}{c} = 2.136\times10^{-27}\ (kg\ m\ s^{-1})$

Chap 4 (4,5,6)

4.4: $E = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) = \frac{\hbar^2\pi^2}{2mL^2}(n_x^2 + n_y^2 + n_z^2)$

$E = 5eV \rightarrow n^2 = n_x^2 + n_y^2 + n_z^2 = 5eV \Big/ \frac{\hbar^2\pi^2}{2m_eL^2} = \frac{5eV \cdot 2m_eL^2}{\hbar^2\pi^2} = 1.33\times10^{21}$

$E_{2,1,1} - E_{1,1,1} = \frac{\hbar^2\pi^2}{2mL^2}(2^2 - 1^2) = \frac{3\hbar^2\pi^2}{2mL^2} = 1.807\times10^{-39}\,J = 1.128\times10^{-20}\,eV$

4.5 L changed to 10^{-9} m,

(1) $n^2 = \frac{E\cdot 2m_eL^2}{\hbar^2\pi^2} = 13.3$

(2) $E_{2,1,1} - E_{1,1,1} = \frac{3\hbar^2\pi^2}{2mL^2} = 1.807\times10^{-19}\,J = 1.128\ eV$

4.6) "observe clear energy discretization" $\rightarrow$ $\frac{\hbar^2\pi^2}{2mL^2}$ is comparable to E

$\frac{\hbar^2\pi^2}{2mL^2} \sim E = 2.5\,eV \implies L \sim \sqrt{\frac{\hbar^2\pi^2}{2m_eE}} = 0.3878\times10^{-9}\,m = 0.3878\,nm$

Alternatively:

$L \sim \lambda = \frac{h}{\sqrt{2m_eE}} = 0.776\,nm$

For proton:

$L_p \sim \sqrt{\frac{\hbar^2\pi^2}{2m_pE}} = 9.05\times10^{-12}\,m = 9.05\times10^{-3}\,nm$

or $L_p \sim \lambda_p = \frac{h}{\sqrt{2m_pE}} = 1.81\times10^{-11}\,m = 1.81\times10^{-2}\,nm$

Chap 7

7.2 $k_BT \ll \frac{e^2}{2C}$

$C = 0.5\times10^{-16}\,F \qquad T \ll \frac{e^2}{k_B\cdot 2C} = 1859\,K$

$C = 1.2\times10^{-12}\,F \qquad T \ll \frac{e^2}{k_B\cdot 2C} = 7.746\times10^{-4}\,K$

7.14: (a) $\lambda = \frac{h}{\sqrt{2m_eE}} = \frac{1240}{\sqrt{2\times5\times0.511\times10^6}} = \frac{1240}{\sqrt{511}}\times\frac{1}{100} = \frac{12.4}{\sqrt{511}} = 5.485\times10^{-10}\,m = 0.5485\,nm$

(b) ernergy different betweence two levels should at least comparable (not too smaller than E

$\rightarrow \frac{\hbar^2\pi^2}{2mL^2} \sim E, \quad L \sim \frac{\lambda}{2} = 0.274\,nm$

(students who give answer like $L\sim\lambda$ or $L \lesssim \lambda$ are also correct)

(c) ~~Yes, to observe clear coulomb blockade, number in quantum dot, engergy state~~

7.14 (c): ~~to have discrete number of electrons on the island~~ energy gap between two levels must be of the size $\frac{e^2}{2C}$ (charging E) (and much larger than thermal energy k_BT), then "coulomb staircase" could be observed. ~~otherwise, for~~

Otherwise, coulomb blockade can be observed as long as certain conditions for T, R_t are satisfied.

So no, it's not necessary to require energy level quantized for normal coulomb blockade, but it's necessary for observing "coulomb staircase" in quantum dot circuit.

Not Necc. Coulomb gap is there even if $L \to \infty$ $\Rightarrow \Delta E \to 0$

8.1:

$$N_{E \le E_n} = \cdot V_{\text{sphere of } n} = \cdot \frac{4\pi n^3}{3}$$ as $n_{x,y,z} = 0, \pm 1, \pm 2 \ldots$ so no "1/8" factor here.

$$E_n = \frac{2\hbar^2\pi^2}{m_e^* L^2} n^2$$

$$\rightarrow \quad N_{E \le E_n} = \cdot \frac{4\pi}{3} \cdot \left(\frac{E \cdot m_e^* L^2}{2\hbar^2 \pi^2} \right)^{\frac{3}{2}}$$

$$N(E) = \frac{d N_{E\le E_n}(E)}{dE} = \frac{4\pi}{3} \cdot \frac{3}{2} \cdot \left(\frac{m_e^* L^2}{2\hbar^2\pi^2} \right)^{\frac{3}{2}} E^{\frac{1}{2}}$$

$$= 2\pi \cdot \frac{m_e^{*\frac{3}{2}} L^3}{2^{\frac{3}{2}} h^3 \pi^3} E^{\frac{1}{2}}$$

consider spin degeneracy:

$$N'(E) = 2 * N(E)_{\text{above}}$$

$$= 4\pi \cdot \frac{m_e^{*\frac{3}{2}} L^3}{2^{\frac{3}{2}} h^3 \pi^3} E^{\frac{1}{2}} = \frac{2^{\frac{1}{2}} m_e^{*\frac{3}{2}} L^3}{\pi^2 h^3} E^{\frac{1}{2}}$$

set $L=1$ and replace E with $E - V_0$

we have same formula as (8.6):

$$N(E) = \frac{2^{\frac{1}{2}} \cdot m^{*\frac{3}{2}}}{\pi^2 h^3} (E - V_0)^{\frac{1}{2}}$$