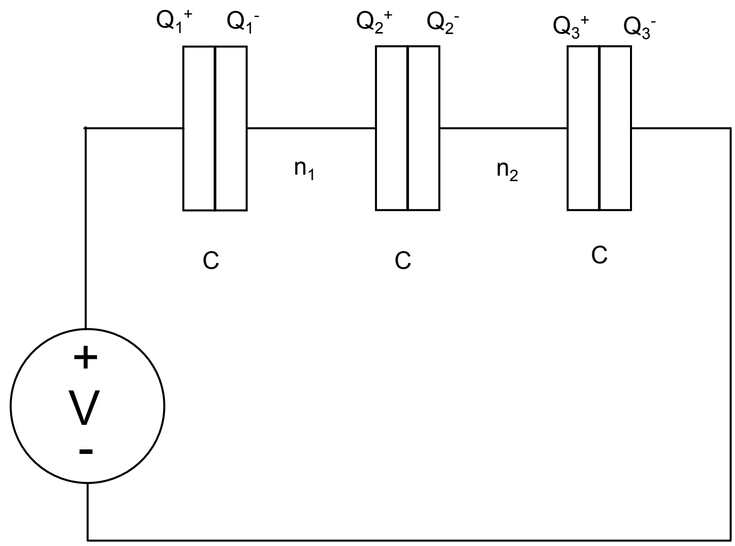


Definitions:
C is the capacitance of each capacitor.
(They are all the same.)
n_1 is the number of excess electrons on island 1.
n_2 is the number of excess electrons on island 2.

$n_{1b} \equiv$ # excess electrons on island 1 before the tunnel event
$n_{1a} \equiv$ # excess electrons on island 1 after the tunnel event
$n_{2b} \equiv$ # excess electrons on island 2 before the tunnel event
$n_{2a} \equiv$ # excess electrons on island 2 after the tunnel event

$$Q_{before} \equiv \int_{-\infty}^{t_{before}} I(t')dt'$$

where I(t') is the current flowing out of the positive terminal of the battery.

$$Q_{after} \equiv \int_{-\infty}^{t_{after}} I(t')dt'$$

where I(t') is the current flowing out of the positive terminal of the battery.

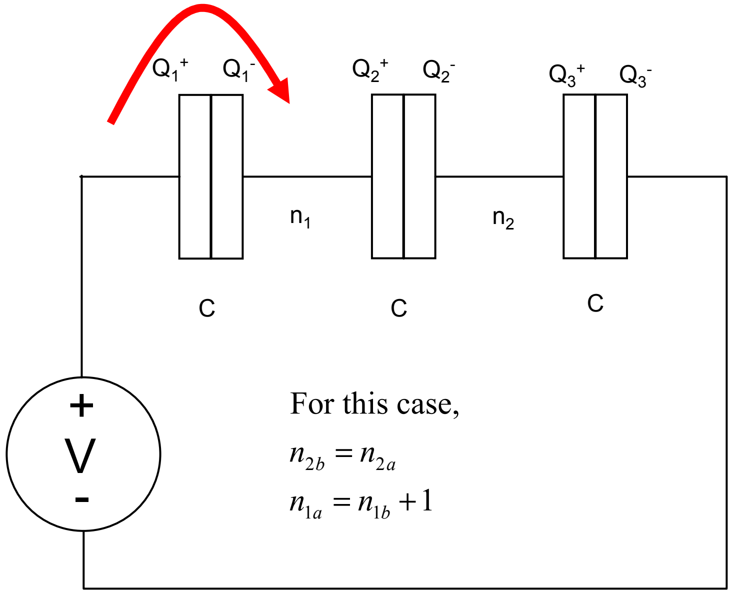
My calculations show that:

$$Q_1 = \frac{1}{3}\left(2en_1 + en_2 + CV\right)$$

$$Q_2 = \frac{1}{3}\left(-en_1 + en_2 + CV\right)$$

$$Q_3 = \frac{1}{3}\left(-en_1 - 2en_2 + CV\right)$$

$$E = \frac{Q_1^2}{2C} + \frac{Q_2^2}{2C} + \frac{Q_3^2}{2C} = \frac{1}{6C}\left[2e^2n_1^2 + 2e^2n_2^2 + (CV)^2 + e^2n_1n_2\right]$$



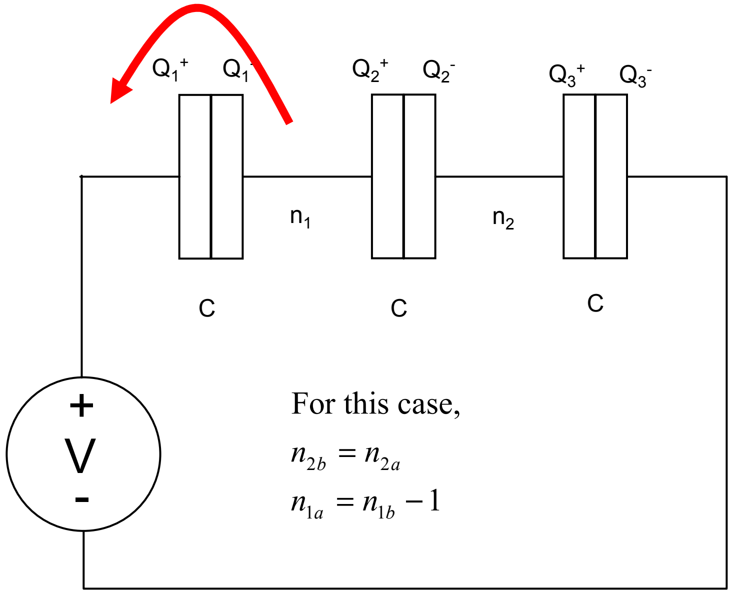
$$\Delta G = G_{before} - G_{after} = \left\{E_{before} - E_{after}\right\} - V\left\{Q_{before} - Q_{after}\right\}$$

$$\left\{E_{before} - E_{after}\right\}$$
$$= \frac{1}{6C}\left[2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}\right] - \frac{1}{6C}\left[2e^2 n_{1a}^2 + 2e^2 n_{2a}^2 + (CV)^2 + e^2 n_{1a} n_{2a}\right]$$
$$= \frac{e^2}{6C}\left[2n_{1b}^2 + n_{1b}n_{2b}\right] - \frac{e^2}{6C}\left[2n_{1a}^2 + n_{1a}n_{2a}\right] = \frac{e^2}{6C}\left[2n_{1b}^2 + n_{1b}n_{2b} - 2(n_{1b}+1)^2 - (n_{1b}+1)n_{2b}\right]$$
$$= \frac{e^2}{6C}\left[2n_{1b}^2 + n_{1b}n_{2b} - 2(n_{1b}^2 + 2n_{1b} + 1) - (n_{1b}+1)n_{2b}\right] = \frac{e^2}{6C}\left[2n_{1b}^2 + n_{1b}n_{2b} - 2n_{1b}^2 - 4n_{1b} - 2 - n_{1b}n_{2b} - n_{2b}\right]$$
$$= \frac{e^2}{6C}\left[-4n_{1b} - 2 - n_{2b}\right]$$

$$\left\{Q_{before} - Q_{after}\right\} = \int_{-\infty}^{t_{before}} I(t')dt' - \int_{-\infty}^{t_{after}} I(t')dt' = -\int_{t_{before}}^{t_{after}} I(t')dt'$$
$$= -\int_{t_{before}}^{t_{after}} I_{tunnel}(t')dt' - \int_{t_{before}}^{t_{after}} I_{polarization}(t')dt'$$
$$= e - \int_{t_{before}}^{t_{after}} I_{polarization}(t')dt' = e + (Q_{1before} - Q_{1after})$$

$$(Q_{1before} - Q_{1after}) = \frac{1}{3}\left(2en_{1b} + en_{2b} + CV\right) - \frac{1}{3}\left(2en_{1a} + en_{2a} + CV\right) = \frac{1}{3}\left(2en_{1b}\right) - \frac{1}{3}\left(2en_{1a}\right)$$
$$= \frac{2e}{3}(n_{1b} - n_{1a}) = \frac{2e}{3}\left(n_{1b} - (n_{1b}+1)\right) = -\frac{2e}{3}$$

$$\Delta G = \frac{e^2}{6C}\left[-4n_{1b} - 2 - n_{2b}\right] - V\left\{\frac{e}{3}\right\}$$



For this case,

$n_{2b} = n_{2a}$

$n_{1a} = n_{1b} - 1$

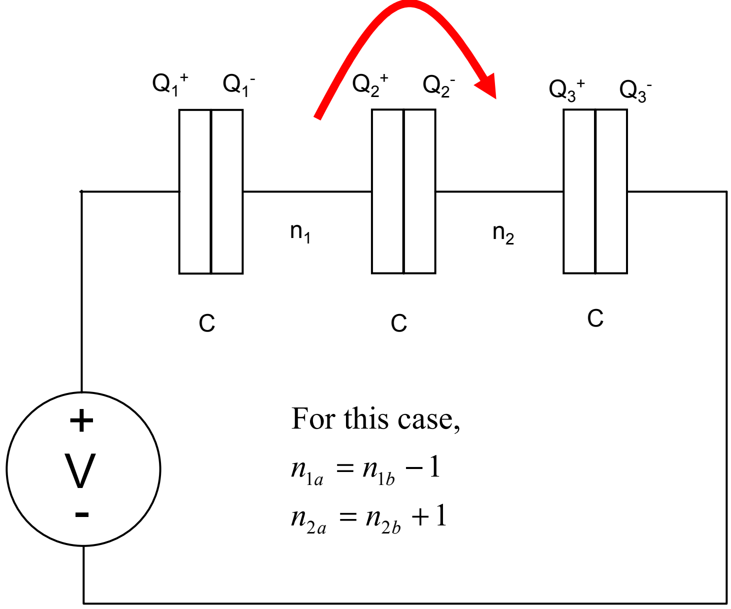
$$\Delta G = G_{before} - G_{after} = \left\{E_{before} - E_{after}\right\} - V\left\{Q_{before} - Q_{after}\right\}$$

$$\begin{aligned}
&\left\{E_{before} - E_{after}\right\} \\
&= \frac{1}{6C}\left[2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}\right] - \frac{1}{6C}\left[2e^2 n_{1a}^2 + 2e^2 n_{2a}^2 + (CV)^2 + e^2 n_{1a} n_{2a}\right] \\
&= \frac{e^2}{6C}\left[2n_{1b}^2 + n_{1b}n_{2b}\right] - \frac{e^2}{6C}\left[2n_{1a}^2 + n_{1a}n_{2a}\right] = \frac{e^2}{6C}\left[2n_{1b}^2 + n_{1b}n_{2b} - 2(n_{1b}-1)^2 - (n_{1b}-1)n_{2b}\right] \\
&= \frac{e^2}{6C}\left[2n_{1b}^2 + n_{1b}n_{2b} - 2(n_{1b}^2 - 2n_{1b} + 1) - (n_{1b}-1)n_{2b}\right] = \frac{e^2}{6C}\left[2n_{1b}^2 + n_{1b}n_{2b} - 2n_{1b}^2 + 4n_{1b} - 2 - n_{1b}n_{2b} + n_{2b}\right] \\
&= \frac{e^2}{6C}\left[+4n_{1b} - 2 + n_{2b}\right]
\end{aligned}$$

$$\begin{aligned}
&\left\{Q_{before} - Q_{after}\right\} = \int_{-\infty}^{t_{before}} I(t')dt' - \int_{-\infty}^{t_{after}} I(t')dt' = -\int_{t_{before}}^{t_{after}} I(t')dt' \\
&= -\int_{t_{before}}^{t_{after}} I_{tunnel}(t')dt' - \int_{t_{before}}^{t_{after}} I_{polarization}(t')dt' \\
&= -e - \int_{t_{before}}^{t_{after}} I_{polarization}(t')dt' = -e + (Q_{1before} - Q_{1after})
\end{aligned}$$

$$\begin{aligned}
&(Q_{1before} - Q_{1after}) = \frac{1}{3}\left(2en_{1b} + en_{2b} + CV\right) - \frac{1}{3}\left(2en_{1a} + en_{2a} + CV\right) = \frac{1}{3}\left(2en_{1b}\right) - \frac{1}{3}\left(2en_{1a}\right) \\
&= \frac{2e}{3}(n_{1b} - n_{1a}) = \frac{2e}{3}\left(n_{1b} - (n_{1b} - 1)\right) = +\frac{2e}{3}
\end{aligned}$$

$$\Delta G = \frac{e^2}{6C}\left[+4n_{1b} - 2 + n_{2b}\right] - V\left\{\frac{-e}{3}\right\}$$



$$\Delta G = G_{before} - G_{after} = \left\{E_{before} - E_{after}\right\} - V\left\{Q_{before} - Q_{after}\right\}$$

$$\left\{E_{before} - E_{after}\right\}$$

$$= \frac{1}{6C}\left[2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}\right] - \frac{1}{6C}\left[2e^2 n_{1a}^2 + 2e^2 n_{2a}^2 + (CV)^2 + e^2 n_{1a} n_{2a}\right]$$

$$= \frac{1}{6C}\left[2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}\right] - \frac{1}{6C}\left[2e^2 (n_{1b}-1)^2 + 2e^2 (n_{2b}+1)^2 + (CV)^2 + e^2 (n_{1b}-1)(n_{2b}+1)\right]$$

$$= \frac{1}{6C}\left[2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}\right] - \frac{1}{6C}\left[2e^2 (n_{1b}^2 - 2n_{1b} + 1) + 2e^2 (n_{2b}^2 + 2n_{2b} + 1) + (CV)^2 + e^2 (n_{1b} n_{2b} + n_{1b} - n_{2b} - 1)\right]$$

$$= \frac{1}{6C}\left[2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}\right] - \frac{1}{6C}\left[2e^2 n_{1b}^2 - 4e^2 n_{1b} + 2e^2 + 2e^2 n_{2b}^2 + 4e^2 n_{2b} + 2e^2 + (CV)^2 + e^2 n_{1b} n_{2b} + e^2 n_{1b} - e^2 n_{2b} - e^2\right]$$

$$= -\frac{1}{6C}\left[-4e^2 n_{1b} + 2e^2 + 4e^2 n_{2b} + 2e^2 + e^2 n_{1b} - e^2 n_{2b} - e^2\right]$$

$$= -\frac{1}{6C}\left[-3e^2 n_{1b} + 3e^2 + 3e^2 n_{2b}\right] = -\frac{3e^2}{6C}\left[-n_{1b} + 1 + n_{2b}\right] = -\frac{e^2}{2C}\left[-n_{1b} + 1 + n_{2b}\right] = \frac{e^2}{2C}\left[n_{1b} - n_{2b} - 1\right]$$

$$\left\{Q_{before} - Q_{after}\right\} = \int_{-\infty}^{t_{before}} I(t')dt' - \int_{-\infty}^{t_{after}} I(t')dt' = -\int_{t_{before}}^{t_{after}} I(t')dt'$$

$$= -\int_{t_{before}}^{t_{after}} I_{tunnel}(t')dt' - \int_{t_{before}}^{t_{after}} I_{polarization}(t')dt'$$

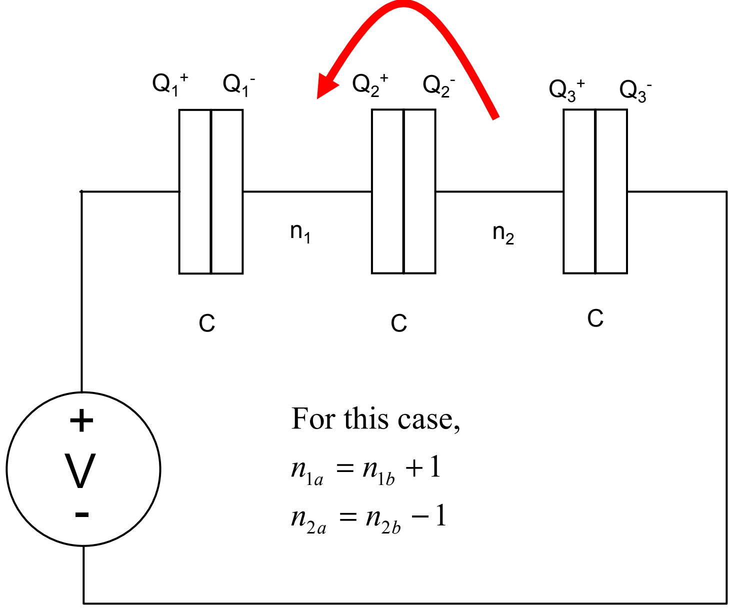
$$= 0 - \int_{t_{before}}^{t_{after}} I_{polarization}(t')dt' = (Q_{1before} - Q_{1after})$$

$$(Q_{1before} - Q_{1after}) = \frac{1}{3}(2en_{1b} + en_{2b} + CV) - \frac{1}{3}(2en_{1a} + en_{2a} + CV) = \frac{1}{3}(2en_{1b} + en_{2b} + CV) - \frac{1}{3}(2e(n_{1b}-1) + e(n_{2b}+1) + CV)$$

$$= \frac{1}{3}(2en_{1b} + en_{2b} + CV) - \frac{1}{3}(2en_{1b} - 2e + en_{2b} + e + CV) = \frac{1}{3}(2en_{1b} + en_{2b} + CV) - \frac{1}{3}(2en_{1b} - e + en_{2b} + CV)$$

$$= \frac{1}{3}(2en_{1b} + en_{2b} + CV - 2en_{1b} + e - en_{2b} - CV) = \frac{+e}{3}$$

$$\Delta G = \frac{e^2}{2C}\left[n_{1b} - n_{2b} - 1\right] - V\left\{\frac{e}{3}\right\}$$



For this case,

$n_{1a} = n_{1b} + 1$

$n_{2a} = n_{2b} - 1$

$$\Delta G = G_{before} - G_{after} = \left\{E_{before} - E_{after}\right\} - V\left\{Q_{before} - Q_{after}\right\}$$

$$\left\{E_{before} - E_{after}\right\}$$

$$= \frac{1}{6C}\left[2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}\right] - \frac{1}{6C}\left[2e^2 n_{1a}^2 + 2e^2 n_{2a}^2 + (CV)^2 + e^2 n_{1a} n_{2a}\right]$$

$$= \frac{1}{6C}\left[2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}\right] - \frac{1}{6C}\left[2e^2 (n_{1b}+1)^2 + 2e^2 (n_{2b}-1)^2 + (CV)^2 + e^2 (n_{1b}+1)(n_{2b}-1)\right]$$

$$= \frac{1}{6C}\left[2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}\right] - \frac{1}{6C}\left[2e^2 (n_{1b}^2 + 2n_{1b} + 1) + 2e^2 (n_{2b}^2 - 2n_{2b} + 1) + (CV)^2 + e^2 (n_{1b} n_{2b} - n_{1b} + n_{2b} - 1)\right]$$

$$= \frac{1}{6C}\left[2e^2 n_{1b}^2 + 2e^2 n_{2b}^2 + (CV)^2 + e^2 n_{1b} n_{2b}\right] - \frac{1}{6C}\left[2e^2 n_{1b}^2 + 4e^2 n_{1b} + 2e^2 + 2e^2 n_{2b}^2 - 4e^2 n_{2b} + 2e^2 + (CV)^2 + e^2 n_{1b} n_{2b} - e^2 n_{1b} + e^2 n_{2b} - e^2\right]$$

$$= -\frac{1}{6C}\left[+4e^2 n_{1b} + 2e^2 - 4e^2 n_{2b} + 2e^2 - e^2 n_{1b} + e^2 n_{2b} - e^2\right]$$

$$= -\frac{1}{6C}\left[+3e^2 n_{1b} + 3e^2 - 3e^2 n_{2b}\right] = -\frac{3e^2}{6C}\left[+n_{1b} + 1 - n_{2b}\right] = -\frac{e^2}{2C}\left[+n_{1b} + 1 - n_{2b}\right] = \frac{e^2}{2C}\left[-n_{1b} + n_{2b} - 1\right]$$

$$\left\{Q_{before} - Q_{after}\right\} = \int_{-\infty}^{t_{before}} I(t')dt' - \int_{-\infty}^{t_{after}} I(t')dt' = -\int_{t_{before}}^{t_{after}} I(t')dt'$$

$$= -\int_{t_{before}}^{t_{after}} I_{tunnel}(t')dt' - \int_{t_{before}}^{t_{after}} I_{polarization}(t')dt'$$

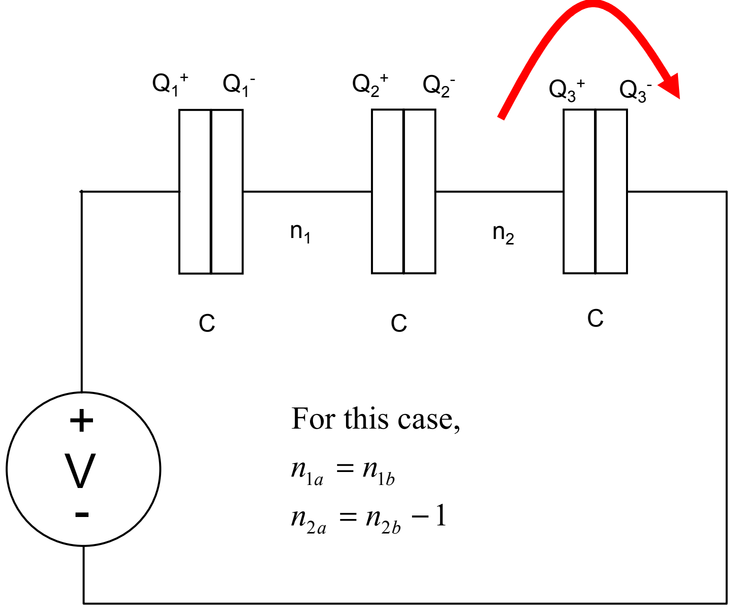
$$= 0 - \int_{t_{before}}^{t_{after}} I_{polarization}(t')dt' = (Q_{1before} - Q_{1after})$$

$$(Q_{1before} - Q_{1after}) = \frac{1}{3}\left(2en_{1b} + en_{2b} + CV\right) - \frac{1}{3}\left(2en_{1a} + en_{2a} + CV\right) = \frac{1}{3}\left(2en_{1b} + en_{2b} + CV\right) - \frac{1}{3}\left(2e(n_{1b}+1) + e(n_{2b}-1) + CV\right)$$

$$= \frac{1}{3}\left(2en_{1b} + en_{2b} + CV\right) - \frac{1}{3}\left(2en_{1b} + 2e + en_{2b} - e + CV\right) = \frac{1}{3}\left(2en_{1b} + en_{2b} + CV\right) - \frac{1}{3}\left(2en_{1b} + e + en_{2b} + CV\right)$$

$$= \frac{1}{3}\left(2en_{1b} + en_{2b} + CV - 2en_{1b} - e - en_{2b} - CV\right) = \frac{-e}{3}$$

$$\Delta G = \frac{e^2}{2C}\left[-n_{1b} + n_{2b} - 1\right] + V\left\{\frac{e}{3}\right\}$$



$$\Delta G = G_{before} - G_{after} = \left\{E_{before} - E_{after}\right\} - V\left\{Q_{before} - Q_{after}\right\}$$

$$\left\{E_{before} - E_{after}\right\}$$

$$= \frac{1}{6C}\left[2e^2n_{1b}^2 + 2e^2n_{2b}^2 + (CV)^2 + e^2n_{1b}n_{2b}\right] - \frac{1}{6C}\left[2e^2n_{1a}^2 + 2e^2n_{2a}^2 + (CV)^2 + e^2n_{1a}n_{2a}\right]$$

$$= \frac{1}{6C}\left[2e^2n_{1b}^2 + 2e^2n_{2b}^2 + (CV)^2 + e^2n_{1b}n_{2b}\right] - \frac{1}{6C}\left[2e^2n_{1b}^2 + 2e^2(n_{2b}-1)^2 + (CV)^2 + e^2n_{1b}(n_{2b}-1)\right]$$

$$= \frac{1}{6C}\left[2e^2n_{1b}^2 + 2e^2n_{2b}^2 + (CV)^2 + e^2n_{1b}n_{2b}\right] - \frac{1}{6C}\left[2e^2n_{1b}^2 + 2e^2(n_{2b}^2 - 2n_{2b} + 1) + (CV)^2 + e^2n_{1b}n_{2b} - e^2n_{1b}\right]$$

$$= \frac{1}{6C}\left[2e^2n_{1b}^2 + 2e^2n_{2b}^2 + (CV)^2 + e^2n_{1b}n_{2b}\right] - \frac{1}{6C}\left[2e^2n_{1b}^2 + 2e^2n_{2b}^2 - 4e^2n_{2b} + 2e^2 + (CV)^2 + e^2n_{1b}n_{2b} - e^2n_{1b}\right]$$

$$= -\frac{1}{6C}\left[-4e^2n_{2b} + 2e^2 + e^2n_{1b}n_{2b} - e^2n_{1b}\right] = \frac{e^2}{6C}\left[4n_{2b} - 2 - n_{1b}n_{2b} + n_{1b}\right]$$

$$\left\{Q_{before} - Q_{after}\right\} = \int_{-\infty}^{t_{before}} I(t')dt' - \int_{-\infty}^{t_{after}} I(t')dt' = -\int_{t_{before}}^{t_{after}} I(t')dt'$$

$$= -\int_{t_{before}}^{t_{after}} I_{tunnel}(t')dt' - \int_{t_{before}}^{t_{after}} I_{polarization}(t')dt'$$

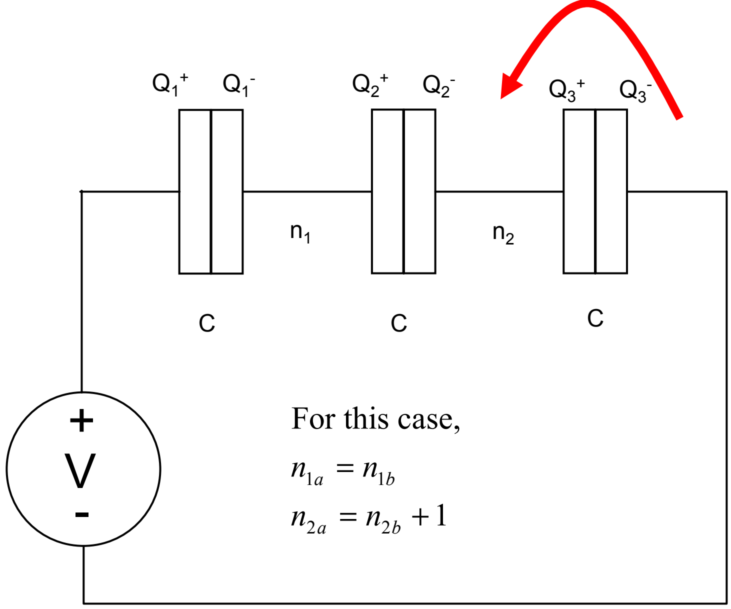
$$= 0 - \int_{t_{before}}^{t_{after}} I_{polarization}(t')dt' = (Q_{1before} - Q_{1after})$$

$$(Q_{1before} - Q_{1after}) = \frac{1}{3}\left(2en_{1b} + en_{2b} + CV\right) - \frac{1}{3}\left(2en_{1a} + en_{2a} + CV\right) = \frac{1}{3}\left(2en_{1b} + en_{2b} + CV\right) - \frac{1}{3}\left(2en_{1b} + e(n_{2b} - 1) + CV\right)$$

$$= \frac{1}{3}\left(2en_{1b} + en_{2b} + CV\right) - \frac{1}{3}\left(2en_{1b} + en_{2b} - e + CV\right)$$

$$= \frac{1}{3}\left(2en_{1b} + en_{2b} + CV - 2en_{1b} + e - en_{2b} - CV\right) = \frac{+e}{3}$$

$$\Delta G = \frac{e^2}{6C}\left[4n_{2b} - 2 - n_{1b}n_{2b} + n_{1b}\right] - V\left\{\frac{e}{3}\right\}$$



For this case,

$n_{1a} = n_{1b}$

$n_{2a} = n_{2b} + 1$

$$\Delta G = G_{before} - G_{after} = \left\{E_{before} - E_{after}\right\} - V\left\{Q_{before} - Q_{after}\right\}$$

$$\left\{E_{before} - E_{after}\right\}$$
$$= \frac{1}{6C}\left[2e^2n_{1b}^2 + 2e^2n_{2b}^2 + (CV)^2 + e^2n_{1b}n_{2b}\right] - \frac{1}{6C}\left[2e^2n_{1a}^2 + 2e^2n_{2a}^2 + (CV)^2 + e^2n_{1a}n_{2a}\right]$$
$$= \frac{1}{6C}\left[2e^2n_{1b}^2 + 2e^2n_{2b}^2 + (CV)^2 + e^2n_{1b}n_{2b}\right] - \frac{1}{6C}\left[2e^2n_{1b}^2 + 2e^2(n_{2b}+1)^2 + (CV)^2 + e^2n_{1b}(n_{2b}+1)\right]$$
$$= \frac{1}{6C}\left[2e^2n_{1b}^2 + 2e^2n_{2b}^2 + (CV)^2 + e^2n_{1b}n_{2b}\right] - \frac{1}{6C}\left[2e^2n_{1b}^2 + 2e^2(n_{2b}^2 + 2n_{2b} + 1) + (CV)^2 + e^2n_{1b}n_{2b} + e^2n_{1b})\right]$$
$$= \frac{1}{6C}\left[2e^2n_{1b}^2 + 2e^2n_{2b}^2 + (CV)^2 + e^2n_{1b}n_{2b}\right] - \frac{1}{6C}\left[2e^2n_{1b}^2 + 2e^2n_{2b}^2 + 4e^2n_{2b} + 2e^2 + (CV)^2 + e^2n_{1b}n_{2b} + e^2n_{1b}\right]$$
$$= -\frac{1}{6C}\left[+4e^2n_{2b} + 2e^2 + e^2n_{1b}n_{2b} + e^2n_{1b}\right] = \frac{e^2}{6C}\left[-4n_{2b} - 2 - n_{1b}n_{2b} - n_{1b}\right]$$

$$\left\{Q_{before} - Q_{after}\right\} = \int_{-\infty}^{t_{before}} I(t')dt' - \int_{-\infty}^{t_{after}} I(t')dt' = -\int_{t_{before}}^{t_{after}} I(t')dt'$$
$$= -\int_{t_{before}}^{t_{after}} I_{tunnel}(t')dt' - \int_{t_{before}}^{t_{after}} I_{polarization}(t')dt'$$
$$= 0 - \int_{t_{before}}^{t_{after}} I_{polarization}(t')dt' = (Q_{1before} - Q_{1after})$$

$$(Q_{1before} - Q_{1after}) = \frac{1}{3}\left(2en_{1b} + en_{2b} + CV\right) - \frac{1}{3}\left(2en_{1a} + en_{2a} + CV\right) = \frac{1}{3}\left(2en_{1b} + en_{2b} + CV\right) - \frac{1}{3}\left(2en_{1b} + e(n_{2b}+1) + CV\right)$$
$$= \frac{1}{3}\left(2en_{1b} + en_{2b} + CV\right) - \frac{1}{3}\left(2en_{1b} + en_{2b} + e + CV\right)$$
$$= \frac{1}{3}\left(2en_{1b} + en_{2b} + CV - 2en_{1b} - e - en_{2b} - CV\right) = \frac{-e}{3}$$

$$\Delta G = \frac{e^2}{6C}\left[-4n_{2b} - 2 - n_{1b}n_{2b} - n_{1b}\right] + V\left\{\frac{e}{3}\right\}$$