NAME_________Solution set________________________

ID______________________________________

EECS 277C: Nanotechnology
Midterm exam
May 9, 2005 6-8:50 PM

1A	1B	1C	2A	2B	Total
/20	/20	/20	/20	/20	/100

Helpful constants for you:

c = 3 10^{8} m/s
e = 1.6 10^{-19} coulombs
h = 6.63 10^{-34} J-s
m = 9.1 10^{-31} kg
k_B = 1.38 10^{-23} J/K

1) Consider a metal nanoparticle of size 10 nm x 10 nm x 10 nm.
The Fermi energy is 10 eV.

A) Find the density of electrons n.

Write down electron density = $1/(10^{-10}\text{m})^3$, no points.
If you don't give units or give the wrong units, no credit at all even if the calculation is correct up to the last step. (Units is a high school concept, this is a graduate class.)

You will be graded by one of two criteria, depending on whether you used the correct or incorrect formula from the lecture notes. (Even though there was an announcement made in class correcting the lecture notes.)
If you use the correct formula for E_F in the lecture notes:

$$E_f = \frac{\hbar^2 3^{2/3}\pi^{4/3}}{2m}\left(\frac{\#\text{ electrons}}{L^3}\right)^{2/3}$$

$$\Rightarrow \left(\frac{\#\text{ electrons}}{L^3}\right) = \left(E_f \frac{2m}{\hbar^2 3^{2/3}\pi^{4/3}}\right)^{3/2} =$$

$$\left(10x1.6x10^{-19}\frac{2x9.1x10^{-31}}{(6.63x10^{-34}/2\pi)^2 3^{2/3}\pi^{4/3}}\right)^{3/2}\frac{1}{m^3} =$$

$$\left(10^{1-19-31+34+34}\frac{1.6x2x9.1x2\pi x2\pi}{6.63x6.63x3^{2/3}x\pi^{4/3}}\right)^{3/2}\frac{1}{m^3} = \left(10^{19}\frac{1.6x2x9.1x2x3.14x2x3.14}{6.63x6.63x2.06x4.58}\right)^{3/2}\frac{1}{m^3}$$

$$= \left(10^{19}\frac{1148}{415}\right)^{3/2}\frac{1}{m^3} = \left(2.7x10^{19}\right)^{3/2}\frac{1}{m^3} = 1.45x10^{29}\frac{1}{m^3}$$

Grading criteria: Write down the correct equation, 5 pts.
Write down correct equation, and correct numbers, but arithmetic mistake: 10 pts.
Correct answer (range 1-2 10^{29} m^{-3}), 20 pts.

If you use the incorrect formula for E_F in the lecture notes:

$$\#\text{ electrons} = L^3\frac{2^{1/2}m^{3/2}}{\pi^2\hbar^{3/2}}\frac{2}{3}E_f^{3/2}\ \text{(incorrect, but in lecture notes)}$$

$$\Rightarrow \frac{\#\text{ electrons}}{L^3} = \left(E_f\frac{2m}{3^{2/3}\hbar\pi^{4/3}}\right)^{3/2} =$$

$$\left(10x1.6x10^{-19}\frac{2x9.1x10^{-31}}{3^{2/3}(6.63x10^{-34}/2\pi)\pi^{4/3}}\right)^{3/2}\frac{1}{m^3} =$$

$$\left(10^{+1-19-31+34}\frac{1.6x2\pi x2x9.1}{3^{2/3}x6.63x\pi^{4/3}}\right)^{3/2}\frac{1}{m^3} =$$

$$\left(10^{-15}\frac{182.9}{62.8}\right)^{3/2}\frac{1}{m^3} = \left(2.9x10^{-15}\right)^{3/2}\frac{1}{m^3} = 1.56x10^{-22}\frac{1}{m^3}$$

Grading criteria: Write down the equation as in lecture notes, 5 pts.
Write down equation as in lecture notes, and correct numbers, but arithmetic mistake: 10 pts.
Answer range 1-2 10^{-22} m^{-3}, 20 pts.

(Note that you need to be consistent in your units. If you use eV for energy units in one place and J-s for units for Planck's constant in another place, you are not consistent in your units and this is incorrect. So you would not get credit for putting in the correct numbers into the formula. This will be marked as "inconsistent units" on your exam.)

B) Find the total number of electrons on the nanoparticle N.

(If you use 2d formulas, no credit. This is a 3d system according to the criteria discussed in class.)
There are 4 ways to do this problem. You will be graded by the way that you use.

Way #1:

$$\text{\# electrons} = \text{density x volume } = 1.45x10^{29}\frac{1}{m^3}x10^{-24}m^3 = 1.45x10^5$$

Grading criteria for way #1: Write down formula N=density x volume, 10 pts.
Put in numbers correctly but units wrong during calculation, 15 pts.
Get correct answer (based on your answer to part A), 20 pts.

Way #2:
If you use the incorrect formula for E_F in the lecture notes:

$$\text{\# electrons} = L^3\frac{2^{1/2}m^{3/2}}{\pi^2\hbar^{3/2}}\frac{2}{3}E_f^{3/2} \text{ (incorrect, but in lecture notes)}$$

$$\Rightarrow (10^{-8}m)^3\left(E_f\frac{2m}{3^{2/3}\hbar\pi^{4/3}}\right)^{3/2} =$$

$$(10^{-8}m)^3\left(10x1.6x10^{-19}\frac{2x9.1x10^{-31}}{3^{2/3}(6.63x10^{-34}/2\pi)\pi^{4/3}}\right)^{3/2}\frac{1}{m^3} =$$

$$(10^{-8}m)^3\left(10^{+1-19-31+34}\frac{1.6x2\pi x2x9.1}{3^{2/3}x6.63x\pi^{4/3}}\right)^{3/2}\frac{1}{m^3} =$$

$$(10^{-8}m)^3\left(10^{-15}\frac{182.9}{62.8}\right)^{3/2}\frac{1}{m^3} = (10^{-8}m)^3\left(2.9x10^{-15}\right)^{3/2}\frac{1}{m^3} = (10^{-8}m)^3 1.56x10^{-22}\frac{1}{m^3} = 1.56x10^{-46}$$

Grading criteria: Write down the equation as in lecture notes, 5 pts.
Write down equation as in lecture notes, and correct numbers, but arithmetic mistake: 10 pts.
Answer range 1-2 10^{-46}, 20 pts.

Way #3:
If you use the correct formula for E_F in the lecture notes:

$$E_f = \frac{\hbar^2 3^{2/3}\pi^{4/3}}{2m}\left(\frac{\text{\# electrons}}{L^3}\right)^{2/3}$$

$$\Rightarrow \text{\# electrons} = L^3\left(E_f\frac{2m}{\hbar^2 3^{2/3}\pi^{4/3}}\right)^{3/2} =$$

$$(10^{-8}m)^3\left(10x1.6x10^{-19}\frac{2x9.1x10^{-31}}{(6.63x10^{-34}/2\pi)^2 3^{2/3}\pi^{4/3}}\right)^{3/2}\frac{1}{m^3} =$$

$$(10^{-8}m)^3\left(10^{1-19-31+34+34}\frac{1.6x2x9.1x2\pi x2\pi}{6.63x6.63x3^{2/3}x\pi^{4/3}}\right)^{3/2}\frac{1}{m^3} = (10^{-8}m)^3\left(10^{19}\frac{1.6x2x9.1x2x3.14x2x3.14}{6.63x6.63x2.06x4.58}\right)^{3/2}\frac{1}{m^3}$$

$$= (10^{-8}m)^3\left(10^{19}\frac{1148}{415}\right)^{3/2}\frac{1}{m^3} = (10^{-8}m)^3\left(2.7x10^{19}\right)^{3/2}\frac{1}{m^3} = (10^{-8}m)^3 1.45x10^{29}\frac{1}{m^3} = 145{,}000$$

Grading criteria: Write down the correct equation as in lecture notes, 5 pts.
Write down equation as in lecture notes, and correct numbers, but arithmetic mistake: 10 pts.
Answer range 1-2 10^5, 20 pts.

Way #4: Derive the relationship between E_F and n from first principles:
Grading criteria: Incorrect but close derivation, 3 points.
Correct derivation: 5 points.
Correct derivation + plug in #s correctly 10 pts.
Correct answer, 20 pts.

C) Find the spacing between energy levels (states) at the Fermi energy. Hint: density of states = number states/energy.

In class, we discussed that the energy levels were vary finely spaced but that they got denser, the higher up in energy you go. We discussed that ρ(E)dE is the number of states with energy between E and E+dE, divided by the volume of the box. Therefore, L^3ρ(E)dE is just the number of states with energy between E and E+dE. So in a given slice dE, there are L^3ρ(E) levels. So the average spacing between levels is just 1/ L^3ρ(E).

Again, there is a mistake in the lecture notes that was corrected in class. But I will still give you credit if you use the lecture notes which have the mistake. So:

Using the correct expression for the density of states:

$$\rho(E) = \frac{2^{1/2} m^{3/2}}{\pi^2 \hbar^3} \cdot E^{1/2}$$

$$\delta E = \frac{1}{L^3 \rho(E)} = \frac{\pi^2 \hbar^3}{L^3 E^{1/2} 2^{1/2} m^{3/2}} = \frac{3.14^2 (6.63x10^{-34}/2\pi)^3}{(10^{-8})^3 (10x1.6x10^{-19})^{1/2} 2^{1/2} (9.1x10^{-31})^{3/2}} J =$$

$$\frac{3.14^2 (6.63/2\pi)^3}{(10x1.6)^{1/2} 2^{1/2} (9.1)^{3/2}} x10^{-34x3} 10^{31x1.5} 10^{19x0.5} 10^{24} J = 0.075x10^{-22} J = 7.5x10^{-24} J = 0.05 meV$$

Grading criteria:
Write down the average spacing between levels is related to 1/ L^3ρ(E), 5 pts.
Write down the average spacing between levels is just 1/ L^3ρ(E), 10 pts.
Write down equation as in lecture notes, and correct numbers, but arithmetic mistake: 15 pts.
Answer range 0.04-0.06 meV, 20 pts.

Using the incorrect (lecture notes) expression for the density of states:

$$\rho(E) = \frac{2^{3/2} m^{3/2}}{\pi^2 \hbar^{3/2}} \cdot E^{1/2}$$

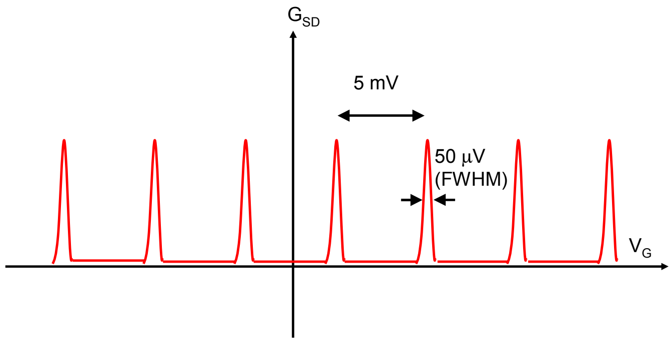
$$\delta E = \frac{1}{L^3 \rho(E)} = \frac{1}{2\hbar^{3/2}} \frac{\pi^2 \hbar^3}{L^3 E^{1/2} 2^{1/2} m^{3/2}} = \frac{1}{2\hbar^{3/2}} \frac{3.14^2 (6.63x10^{-34}/2\pi)^3}{(10^{-8})^3 (10x1.6x10^{-19})^{1/2} 2^{1/2} (9.1x10^{-31})^{3/2}} J =$$

$$\frac{1}{2\hbar^{3/2}} \frac{3.14^2 (6.63/2\pi)^3}{(10x1.6)^{1/2} 2^{1/2} (9.1)^{3/2}} x10^{-34x3} 10^{31x1.5} 10^{19x0.5} 10^{24} J = \frac{1}{2\hbar^{3/2}} 0.075x10^{-22} J = \frac{1}{2\hbar^{3/2}} 7.5x10^{-24} J = \frac{1}{2\hbar^{3/2}} 0.05 meV$$

$$= \frac{1}{2(6.63x10^{-34}/2\pi)^{3/2}} 0.05 meV = 2.3x10^{46} eV$$

Grading criteria: Write down the average spacing between levels is just 1/ L^3ρ(E), 10 pts.
Write down equation as in lecture notes, and correct numbers, but arithmetic mistake: 15 pts.
Answer range(2-3 in prefactor, correct exponent), 20 pts.

For the graph below for a single-electron transistor, find C_G and determine the temperature. Assume $C_1=C_2=C_G$.



A) Criteria:

Write down C=3/2x5mV, 5 pts.
Write down C=e/5mV, 10 pts.
Insert correct #s also but wrong arithmetic, 15 pts.
Correct answer (20-40 aF), 20 pts.
No units, no credit.

$$C_G=\frac{1.6\text{x}10^{-19}}{0.005}\ (\text{F})=32\ \text{aF}=3.2\text{x}10^{-17}\text{F}$$

B) Textbook Ferry Eq. 4.77 gives the functional form for the shape of the curve shown above: (let the peak location be 0 for convenience):

$$\frac{G}{G_{max}}=\cosh^{-2}\left[\frac{e}{3}\frac{V}{2.5kT}\right]$$

Near the peak V close to 0 so need:

$$\cosh[x]\approx 1+x^2$$
$$\cosh^{-2}[x]\approx 1-2x^2$$

At peak, $G/G_{max}=1$. At half max, $G/G_{max}=1/2$. So $V_{halfmax}$ is determined by:

$$\frac{G}{G_{max}}=\frac{1}{2}=\cosh^{-2}\left[\frac{e}{3}\frac{V_{1/2}}{2.5kT}\right]\approx 1-2\left[\frac{e}{3}\frac{V_{1/2}}{2.5kT}\right]^2$$
$$\frac{1}{4}=\left[\frac{e}{3}\frac{V_{1/2}}{2.5kT}\right]^2$$
$$\frac{1}{2}=\left[\frac{e}{3}\frac{V_{1/2}}{2.5kT}\right]$$
$$T=\left[\frac{e}{3}\frac{2V_{1/2}}{2.5k}\right]=\left[\frac{e}{3}\frac{V_{FWHM}}{2.5k}\right]$$

$$T=\frac{1.6\text{x}10^{-19}x0.00005}{3x1.38x10^{-23}}\ (\text{K})=0.19\ \text{K}$$

Criteria: Write down eqn. 4.77, 10 pts.
Write down how to solve problem based on 4.77, 15 pts.
Solve problem (0.18-0.20 K), 20 pts.