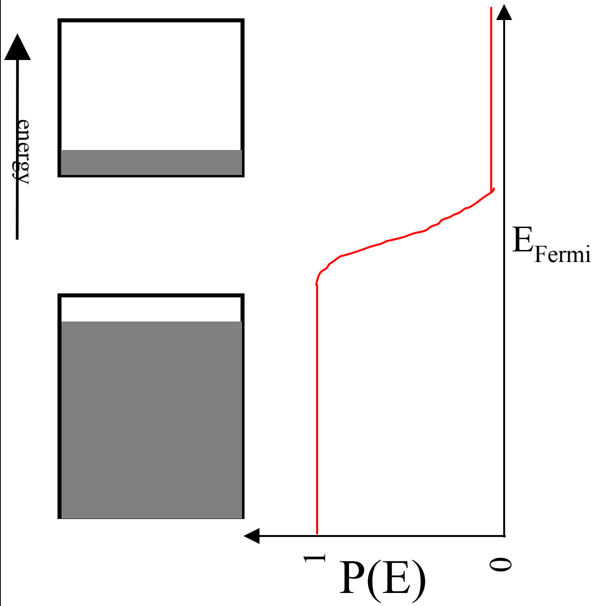
Lecture 13: Semiconductors lasers

- Quasi-Fermi levels
- Optical properties
- pn junctions

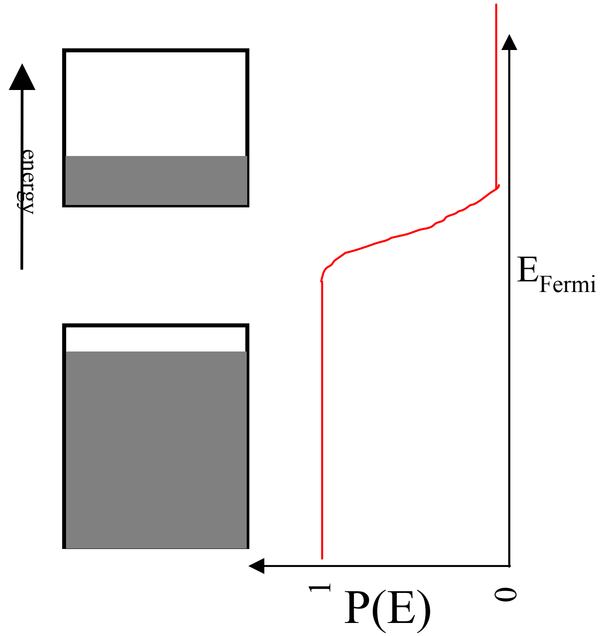
Last time:

Intrinsic:



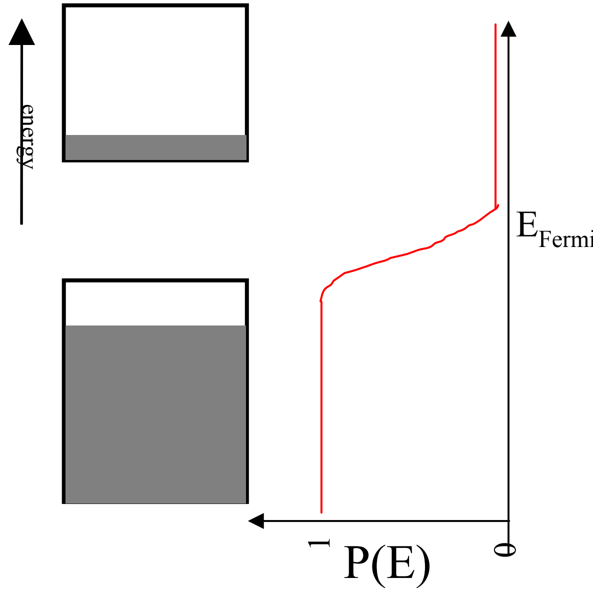
$$n = p$$

n-type:



$$n > p$$

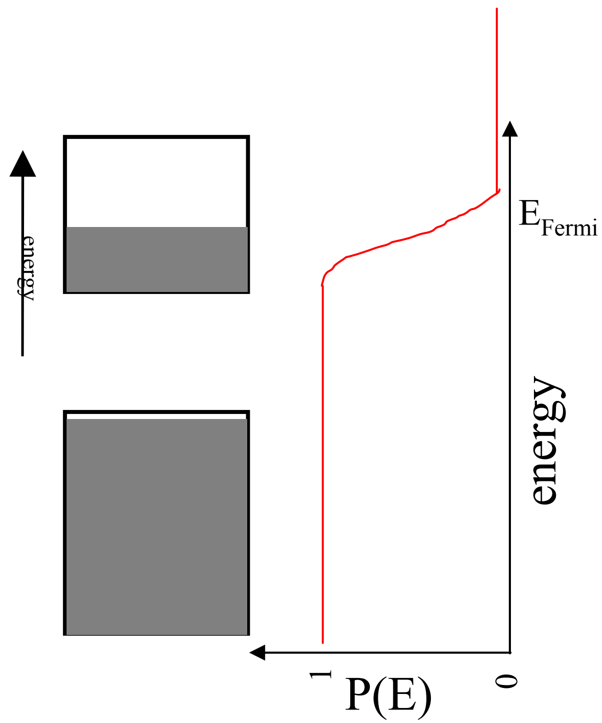
p-type:



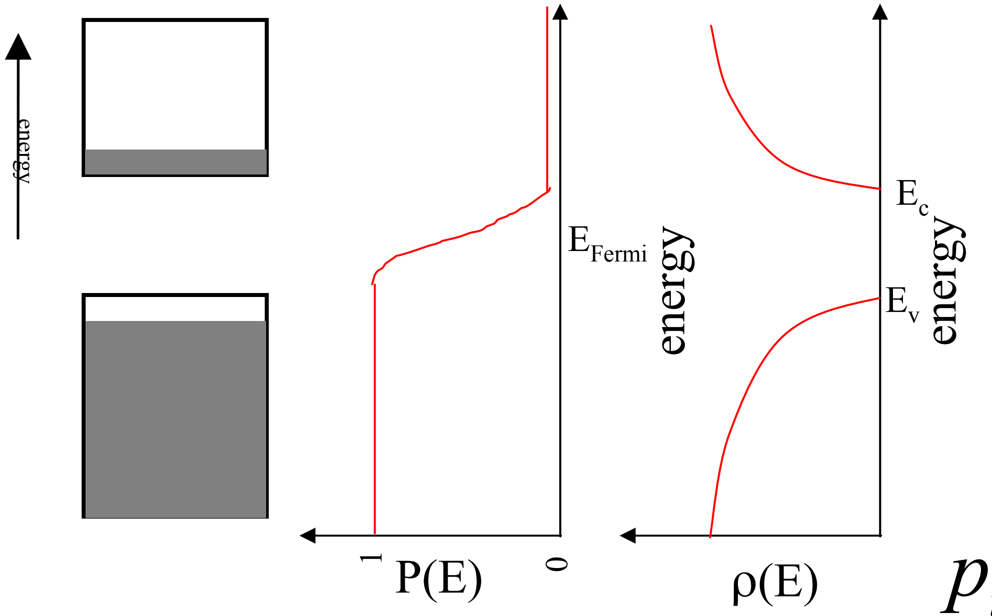
$$n < p$$

n-type:

If Fermi level is all the way up in the conduction band, we call it "degenerately" doped:



Quasi-Fermi levels:



$$n_i = \int_{E_c}^{\infty} P(E)\rho(E)dE$$

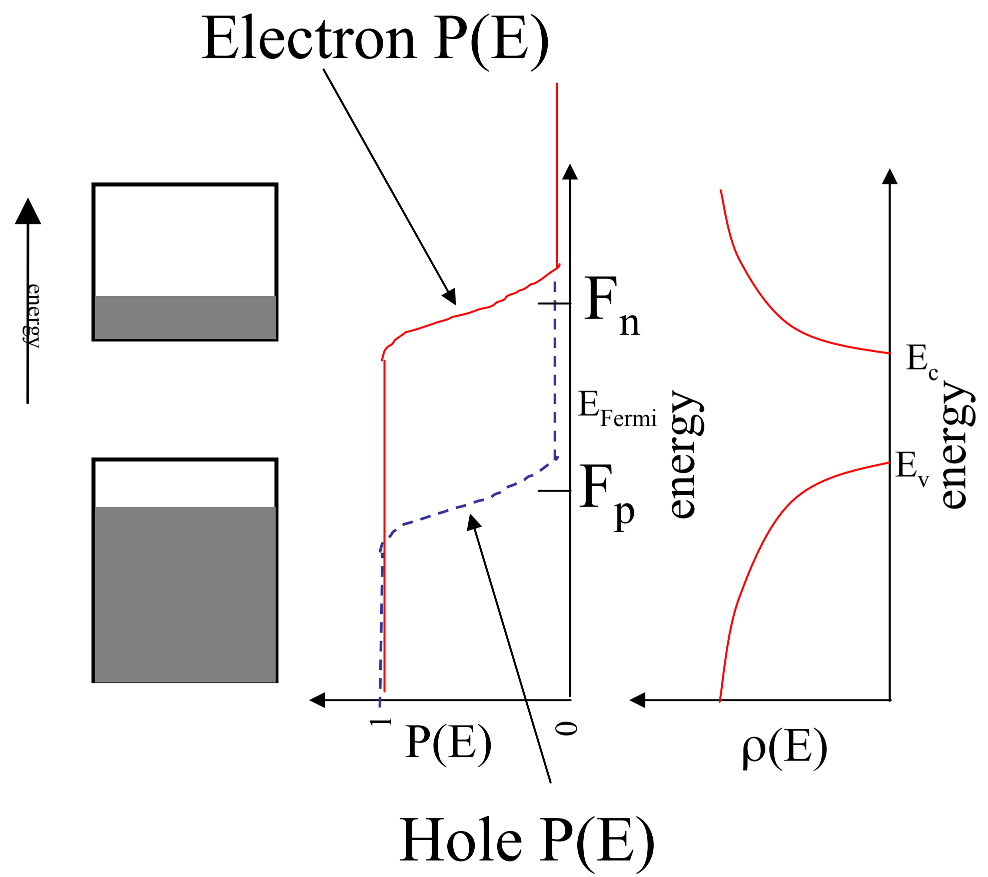
$$P(E) = \frac{1}{e^{(E-E_f)/kT} + 1}$$

$$p_i = \int_{E_c}^{-\infty} [1 - P(E)]\rho(E)dE$$

In equilibrium, E_{Fermi} is the same for n, p calculation.

Out of equilibrium, we use a different value of E_{Fermi} for electrons and holes: F_n and F_p. This is useful later.

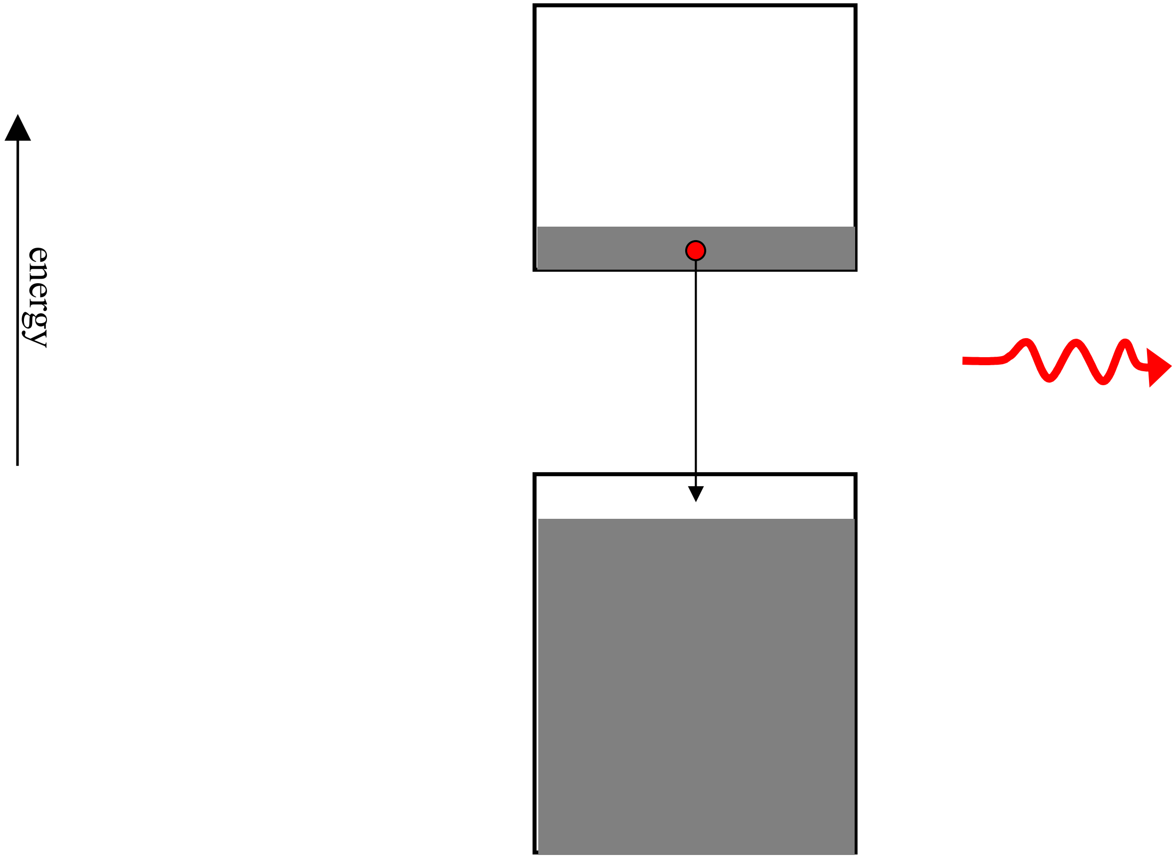
Quasi-Fermi levels:



Quasi-Fermi levels:

- This is NON-EQUILIBRIUM.
- How do we get it? p-n junction diode, to be discussed later.
- For now, assume we have this “population inversion” and derive optical quantities.

Optical transitions:
Spontaneous emission:



This slide only considers energy.
We must also consider MOMENTUM…

Full Schrodinger equation:

Free particle:

$$i\hbar \frac{\partial}{\partial t}\Psi(\vec{r},t) = -\frac{\hbar^2}{2m}\vec{\nabla}^2\Psi(\vec{r},t)$$

Plane wave solutions:

$$\Psi(\vec{r},t) = A \cdot e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

We separated r and t dependence:

$$\Psi(\vec{r},t) = \psi(\vec{r}) \cdot e^{-i\omega t} \qquad \psi(\vec{r}) = Ae^{i\vec{k}\cdot\vec{r}}$$

Gives time independent Schrodinger equation: $\omega = E/\hbar$

$$-\frac{\hbar^2}{2m}\vec{\nabla}^2\psi(\vec{r}) = E \cdot \psi(\vec{r})$$

$$\vec{F} = -\vec{\nabla}V(\vec{r})$$

If the electron feels an external force through potential V(r) then new term:

$$-\frac{\hbar^2}{2m}\vec{\nabla}^2\psi(\vec{r}) + \boxed{V(\vec{r})\psi(\vec{r})} = E \cdot \psi(\vec{r})$$

Full Schrodinger equation:

$$-\frac{\hbar^2}{2m}\vec{\nabla}^2\psi(\vec{r})+V(\vec{r})\psi(\vec{r})=E\cdot\psi(\vec{r})$$

For one electron in a hydrogen atom:

$$V(\vec{r})=-\frac{e^2}{|\vec{r}|}$$

Solutions are not plane waves!

But for many atoms in a crystal,

$$V(\vec{r})=\sum_{atoms}V_{atom}(\vec{r})$$

This is *periodic* in r. Solutions are like plane waves. Bloch's theorem states:

$$\psi(\vec{r})=Ae^{i\vec{k}\cdot\vec{r}}\cdot u(\vec{k},\vec{r})$$

Where $u(\vec{k},\vec{r})$ is periodic in π/a, the lattice wave-vector.
(a is the spacing between atoms in the crystal).

Bloch theorem conclusions:

- If that is all mumbo-jumbo, here's what you need to know:
- Electrons in solid are like free particles (free plane waves) except:

1) Only need to consider wavevectors $k<\pi/a$. (Recall p=hbar k).

2) E vs. k diagram no longer simple.

E vs k:

Free particle:

$$E(k) = \frac{\hbar^2 k^2}{2m}$$

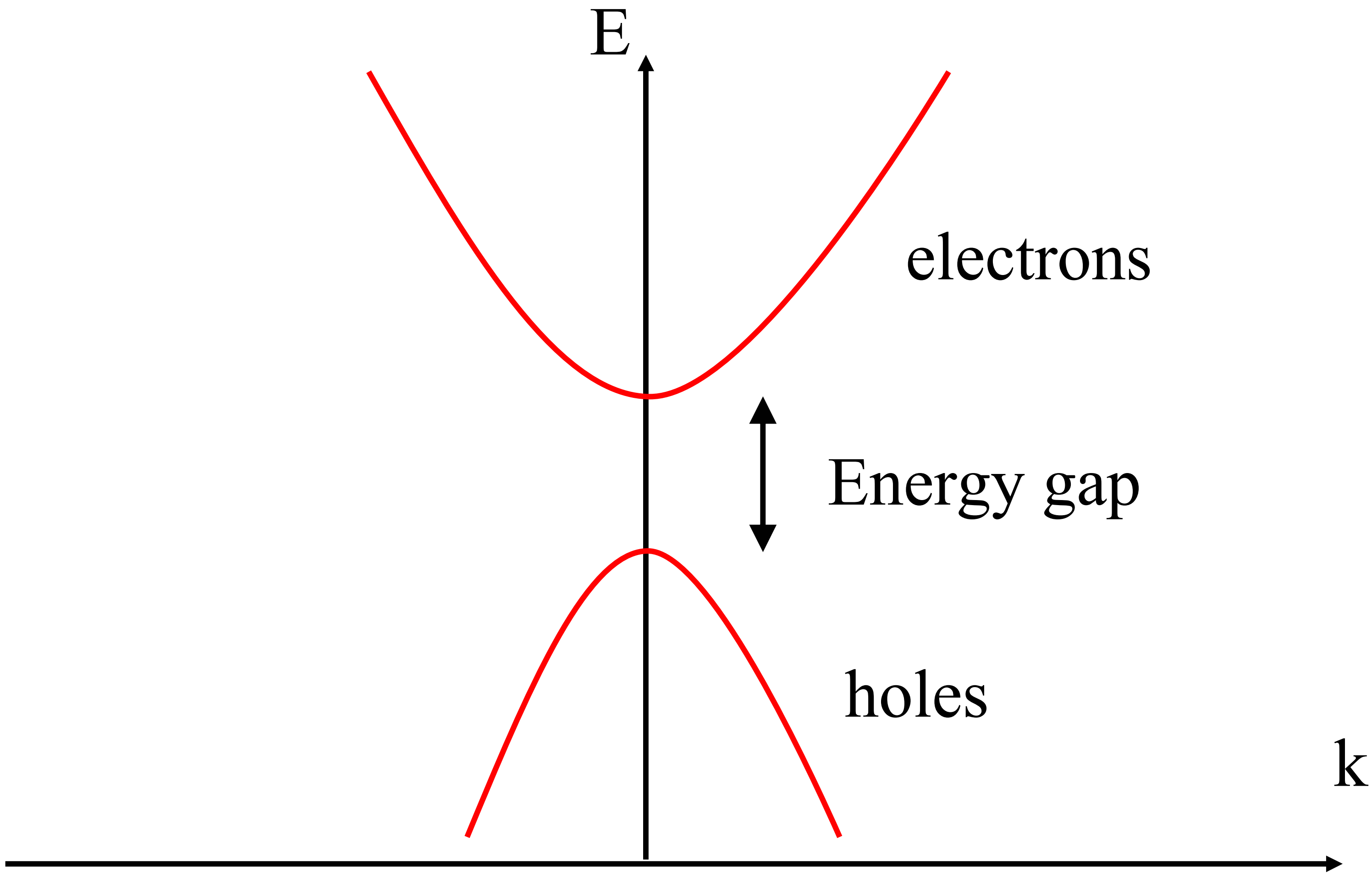
Electron or hole in semiconductor:

$$E(k) = ?$$

Hard to predict, must be measured.

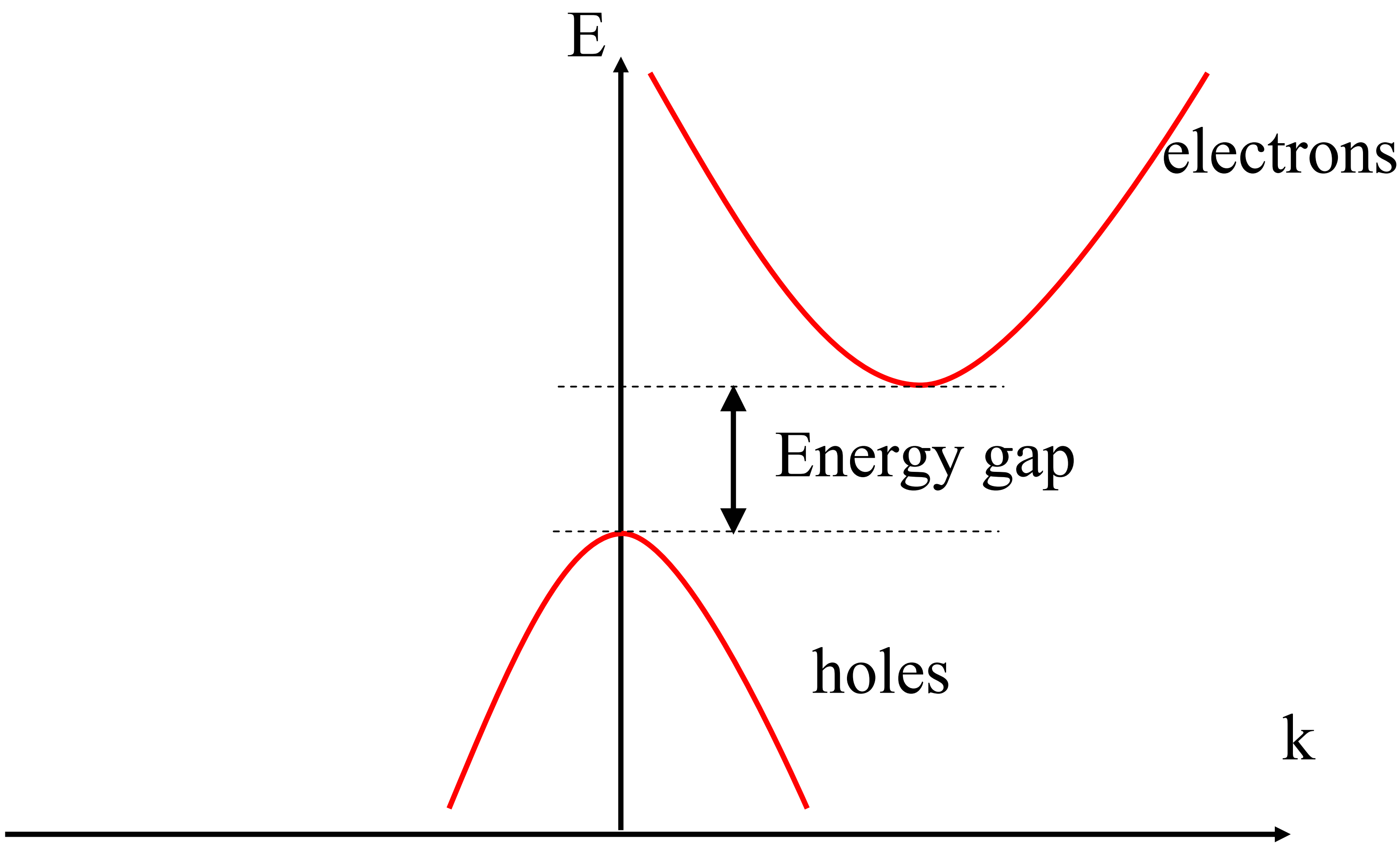
Direct gap material:

E.G. GaAs, InP, InGaAs



Indirect gap material:

E.G. Si



Photons have momentum, too.

Photons

$$E = \hbar\omega$$

$$p = E / c$$

Compared to electrons, photon momentum is very, very small. We will treat it as ZERO.

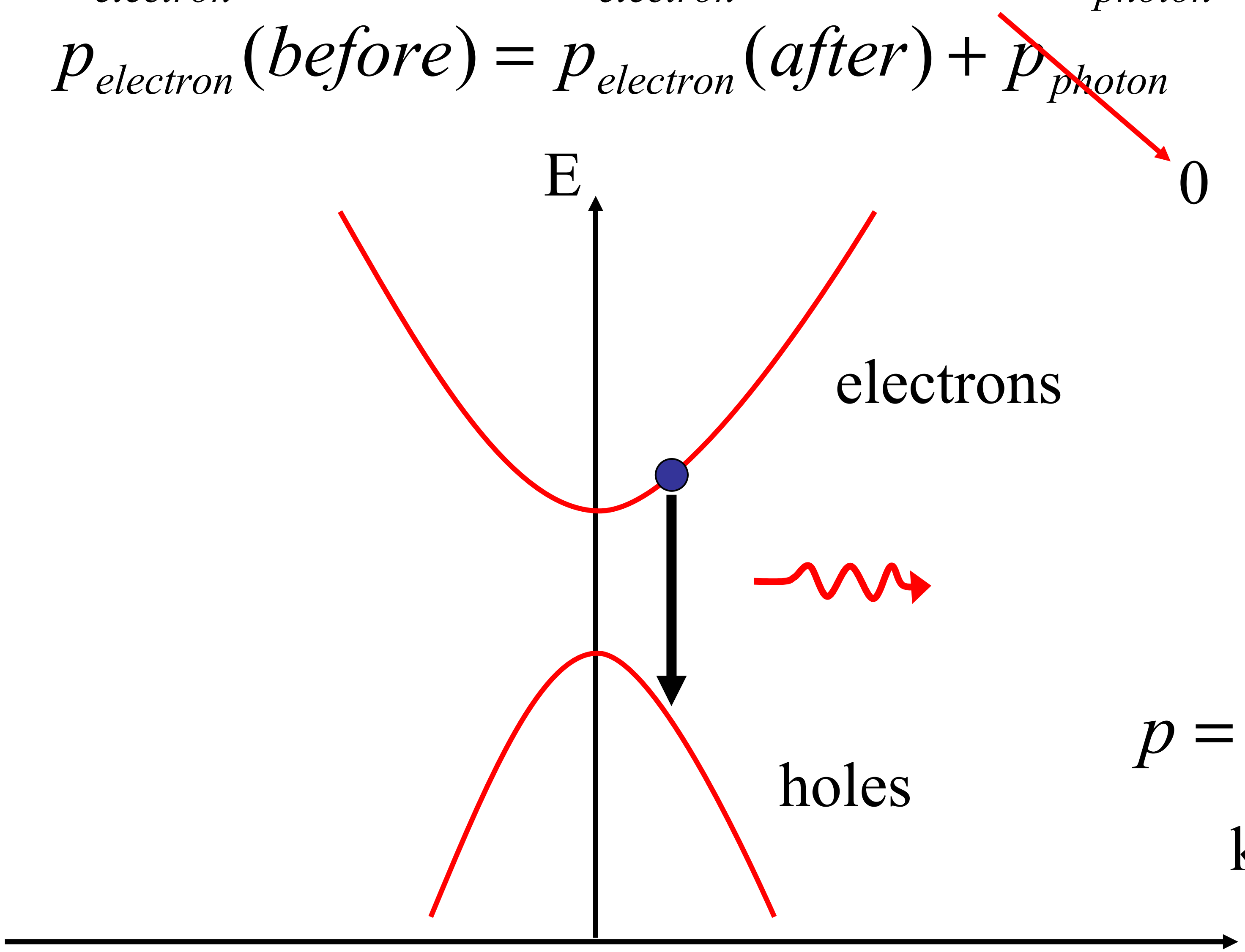
Energy and momentum are always conserved.

Especially momentum.

Spontaneous emission:

$$E_{electron}(before) = E_{electron}(after) + \hbar\omega_{photon}$$

$$p_{electron}(before) = p_{electron}(after) + p_{photon}$$

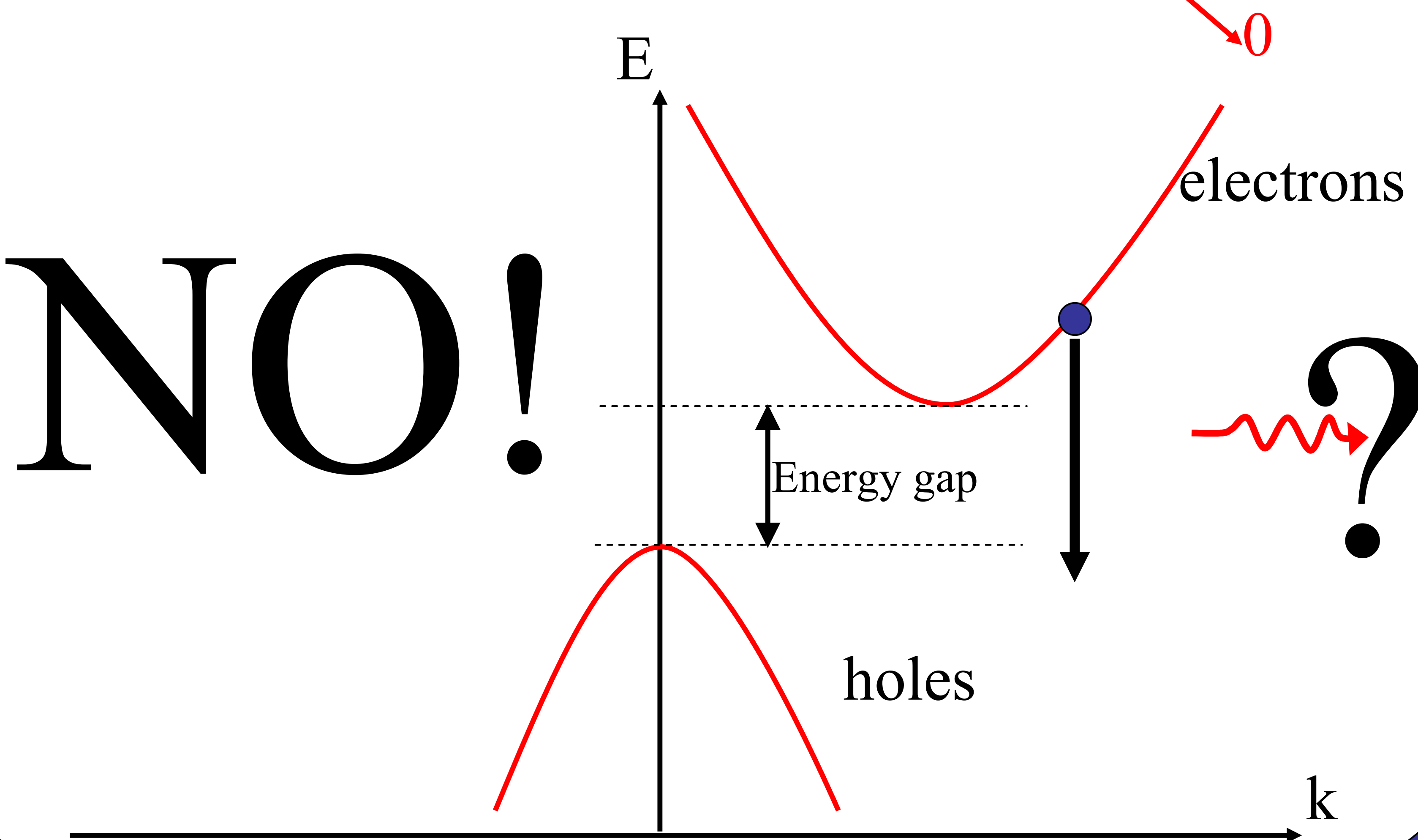


$$p = \hbar k$$

Spontaneous emission:

$$E_{electron}(before) = E_{electron}(after) + \hbar\omega_{photon}$$

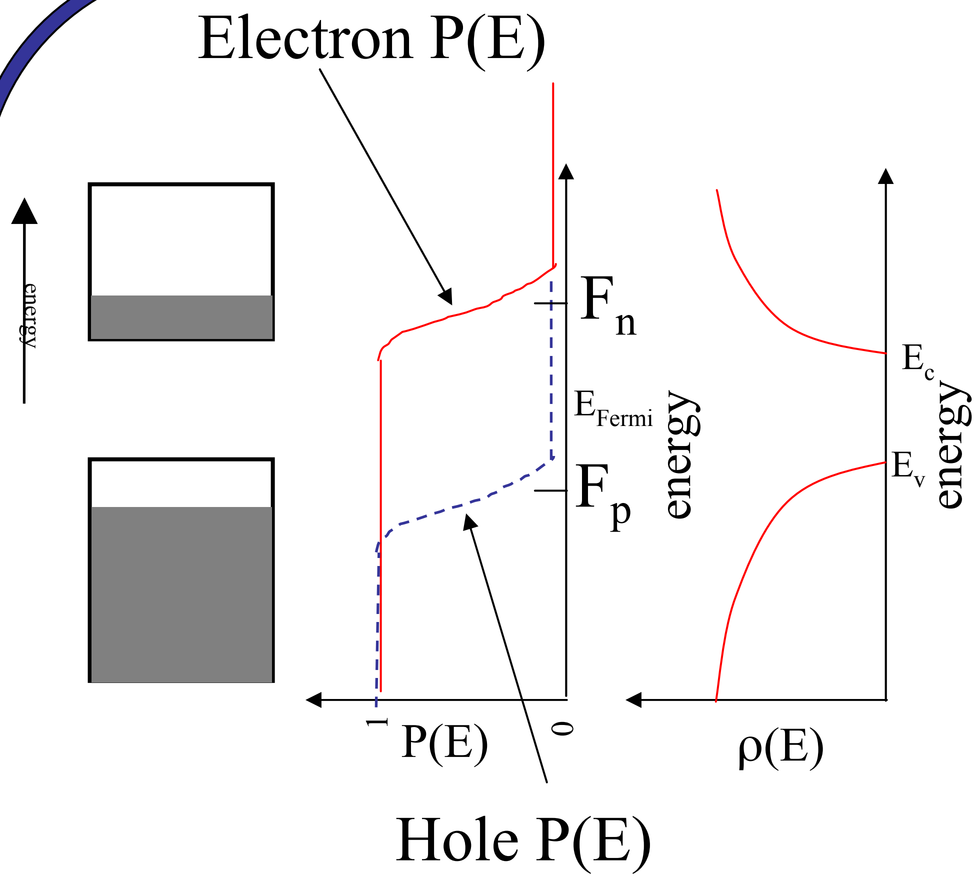
$$p_{electron}(before) = p_{electron}(after) + p_{photon}$$



Direct vs. indirect gap:

- Indirect gap semiconductors (SILICON) are not useful for light emitters such as lasers and LEDs
- Indirect gap semiconductors (GaAs, InP) ARE useful for lasers and LEDs
- Both are useful for optical DETECTORs based on absorption
- These are bulk arguments; silicon nanocrystals do not behave as bulk and this is a hot research topic. See "Optical gain **in** silicon nanocrystals", Pavesi L, Dal Negro L, Mazzoleni C, Franzo G, Priolo F, NATURE 408 (6811): 440-444 NOV 23 2000
- (Opportunity for extra credit)

Quasi-Fermi levels:



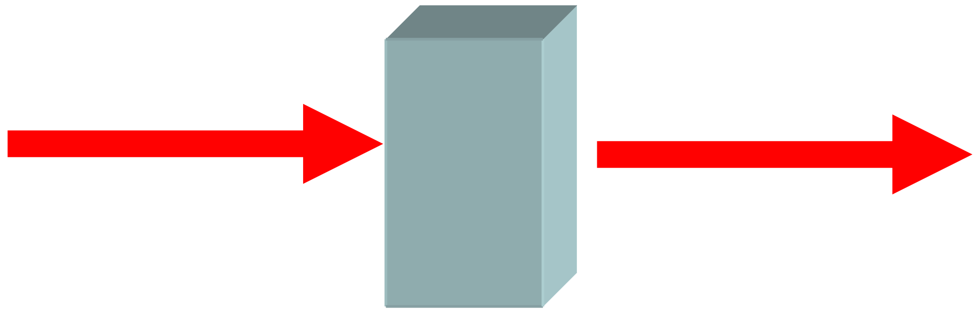
If we achieve this, electrons would spontaneously emit photons and tend to make $F_n=F_p$.

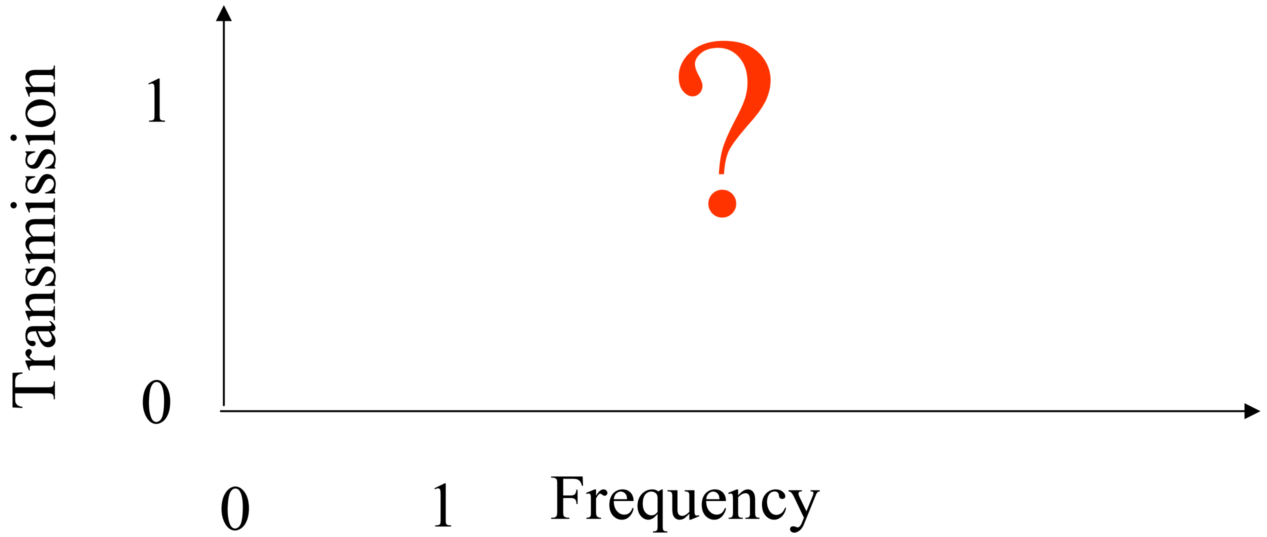
By injecting electrons from the n-side of a p-n diode, we can "pump" electrons into the system, thus maintaining the population inversion.

For now, assume

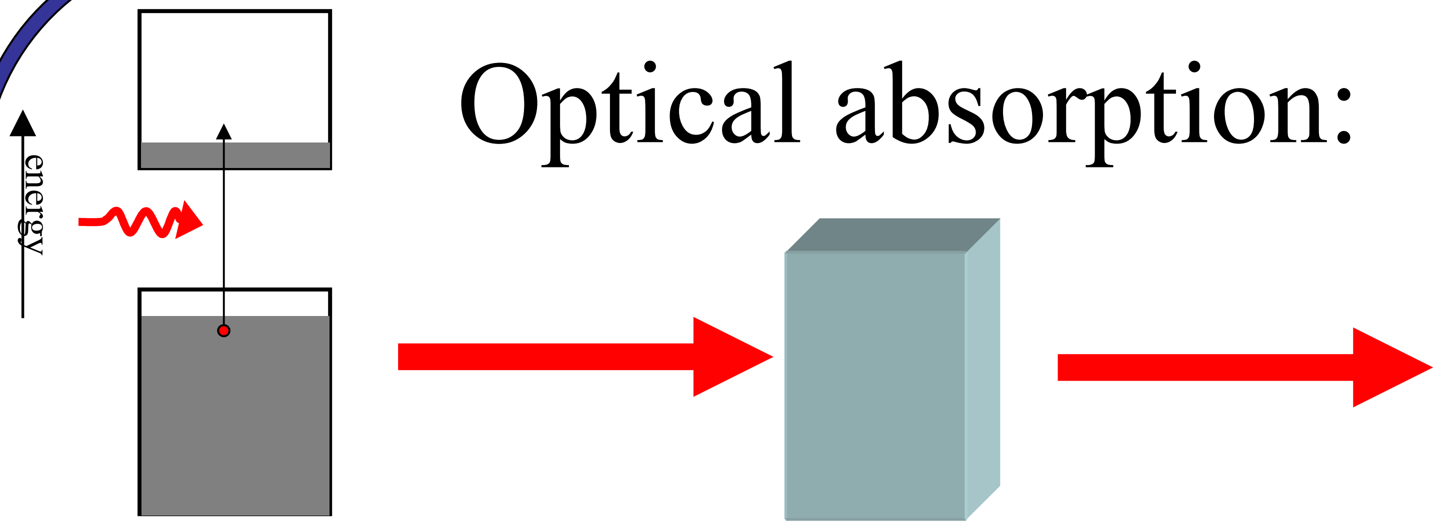
$F_n \neq F_p$.

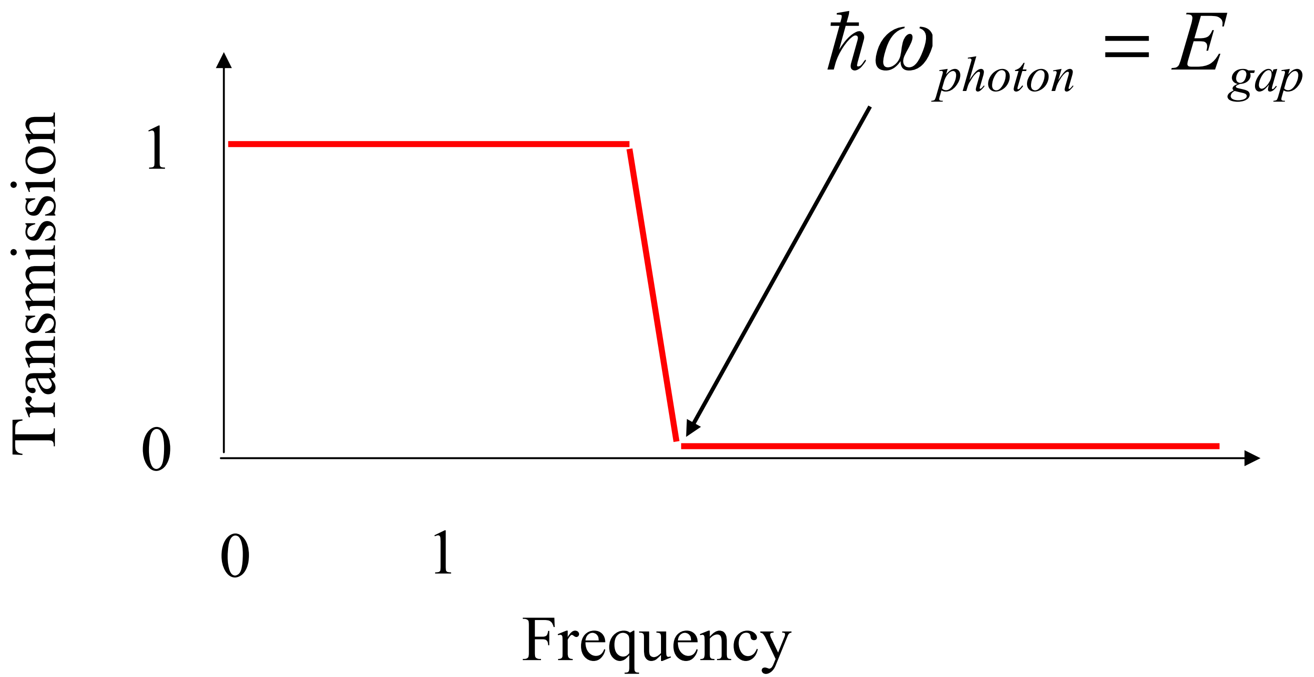
Optical absorption:



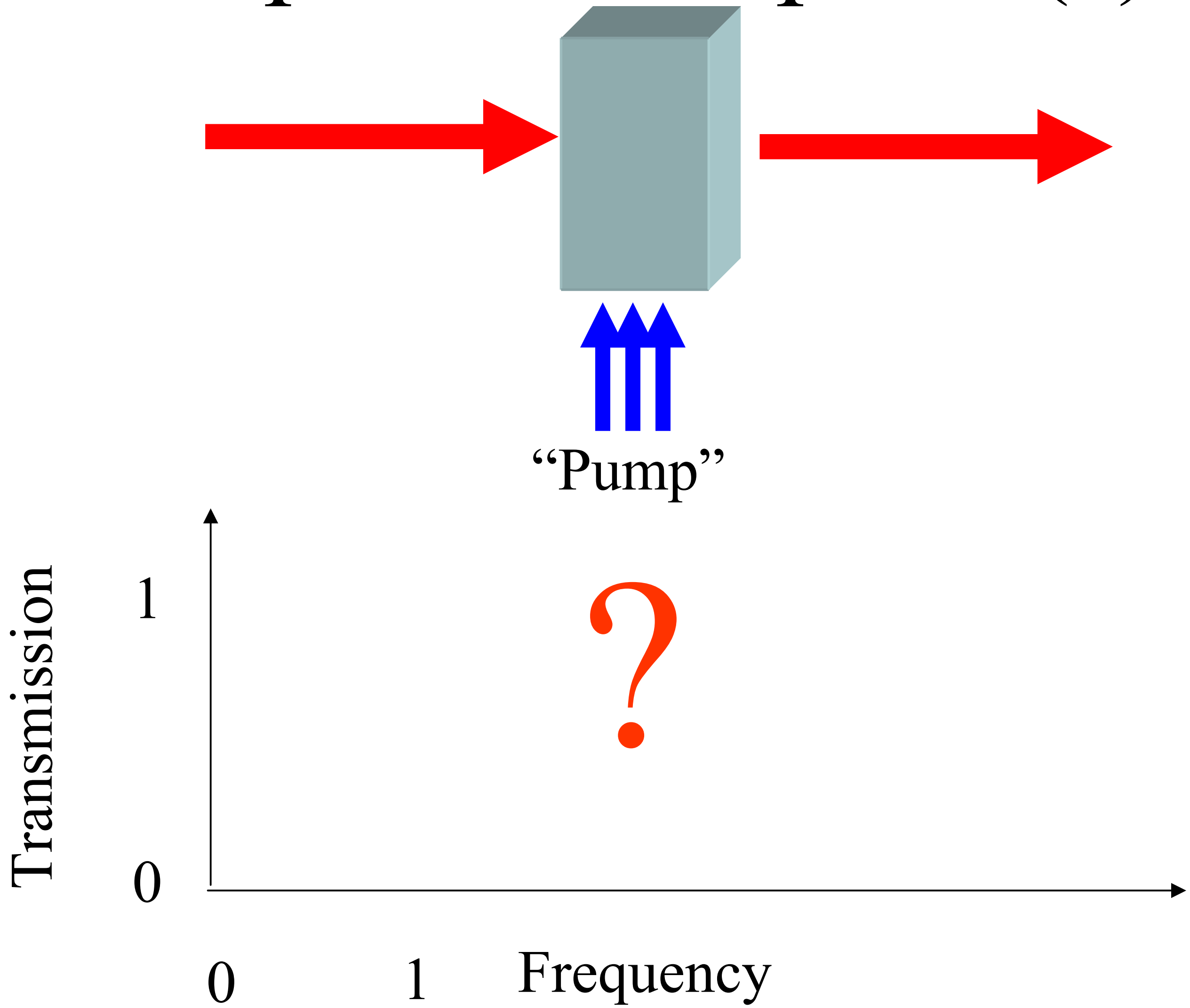


Optical absorption:

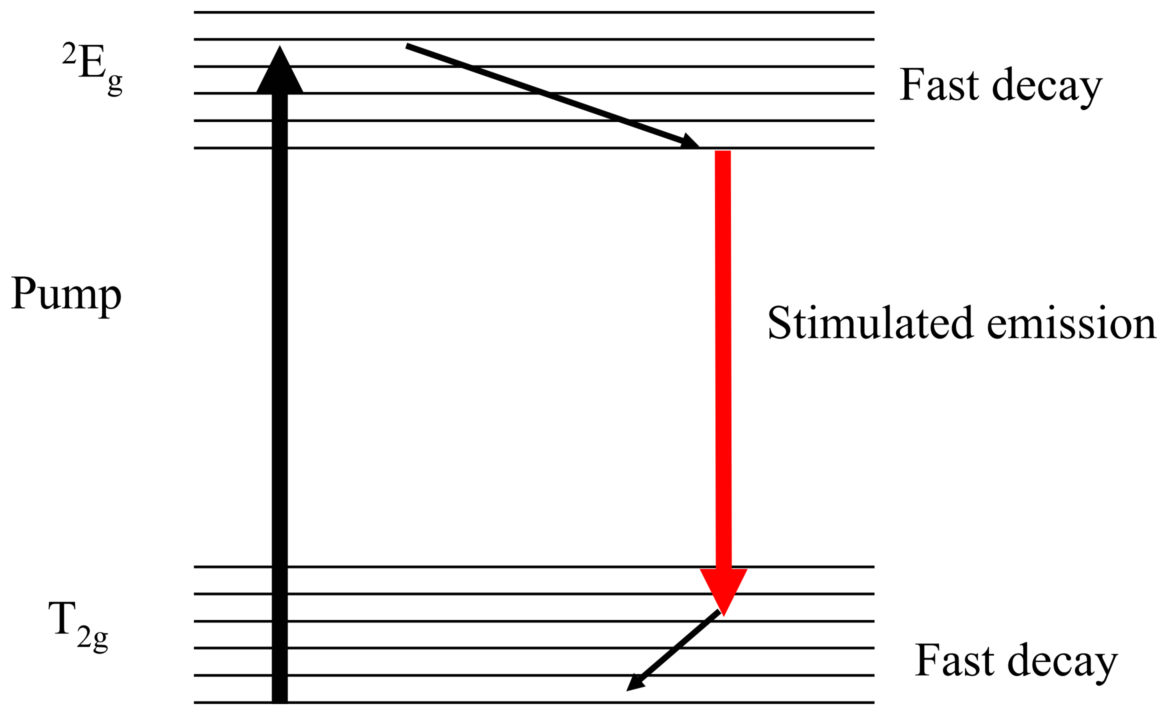




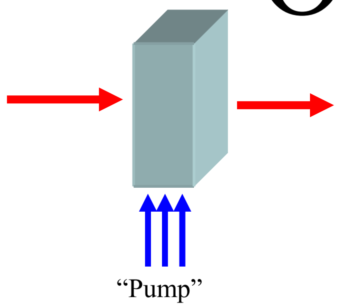
Optical absorption (?):

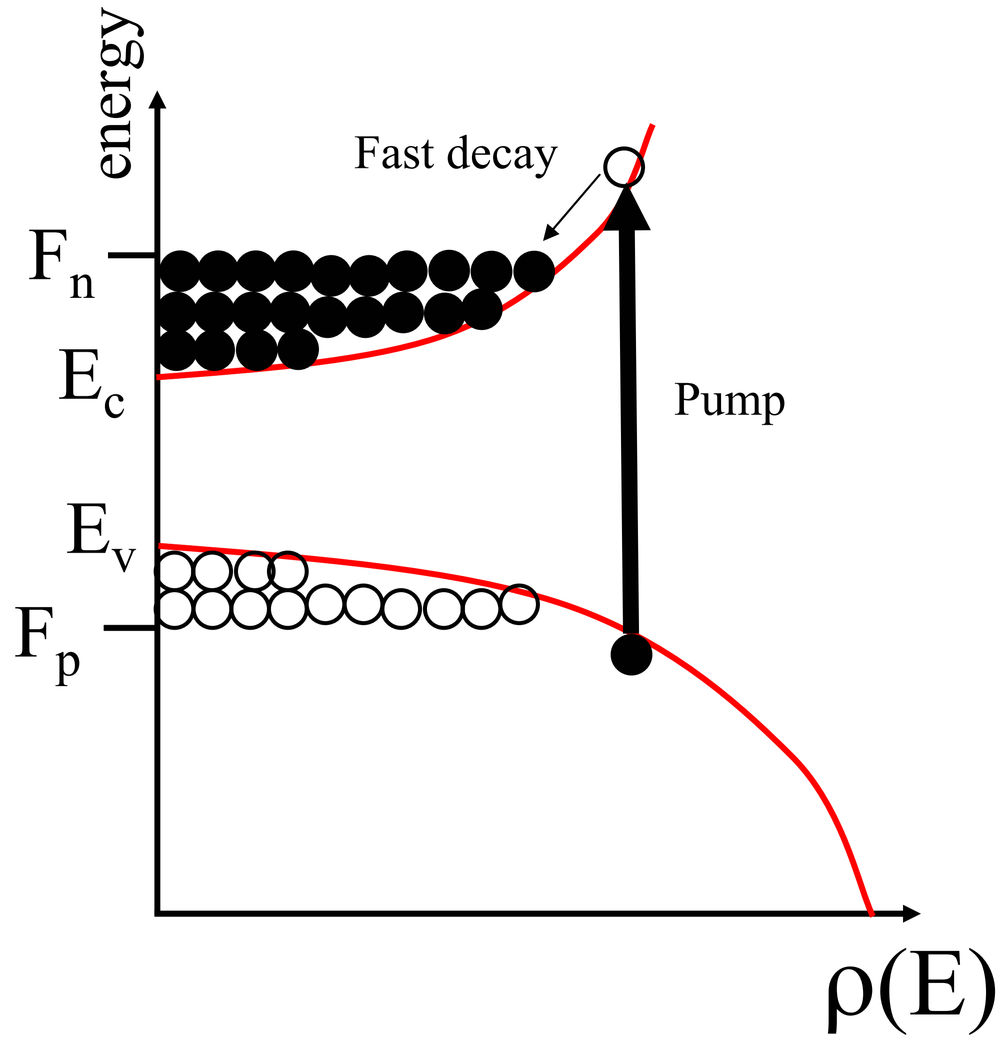


Recall Ti:Sapphire



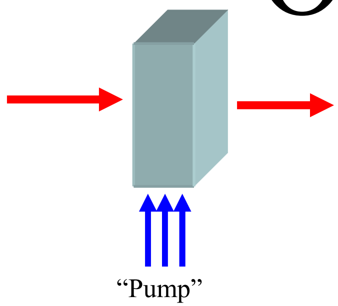
Optical absorption (?):

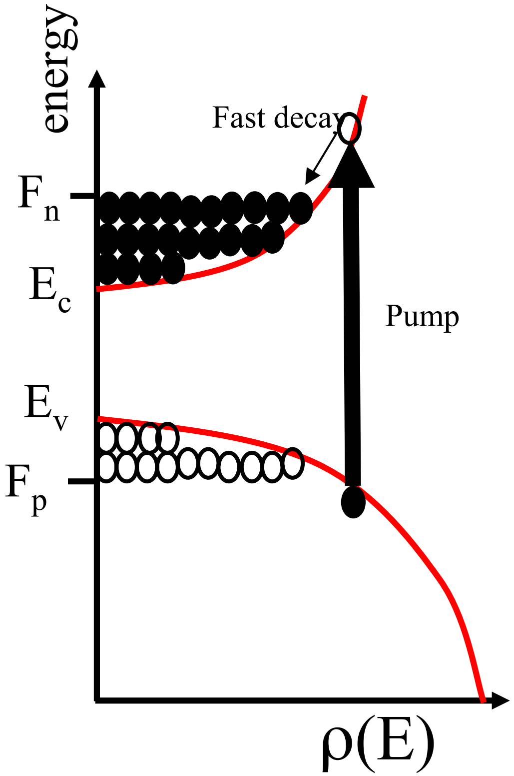


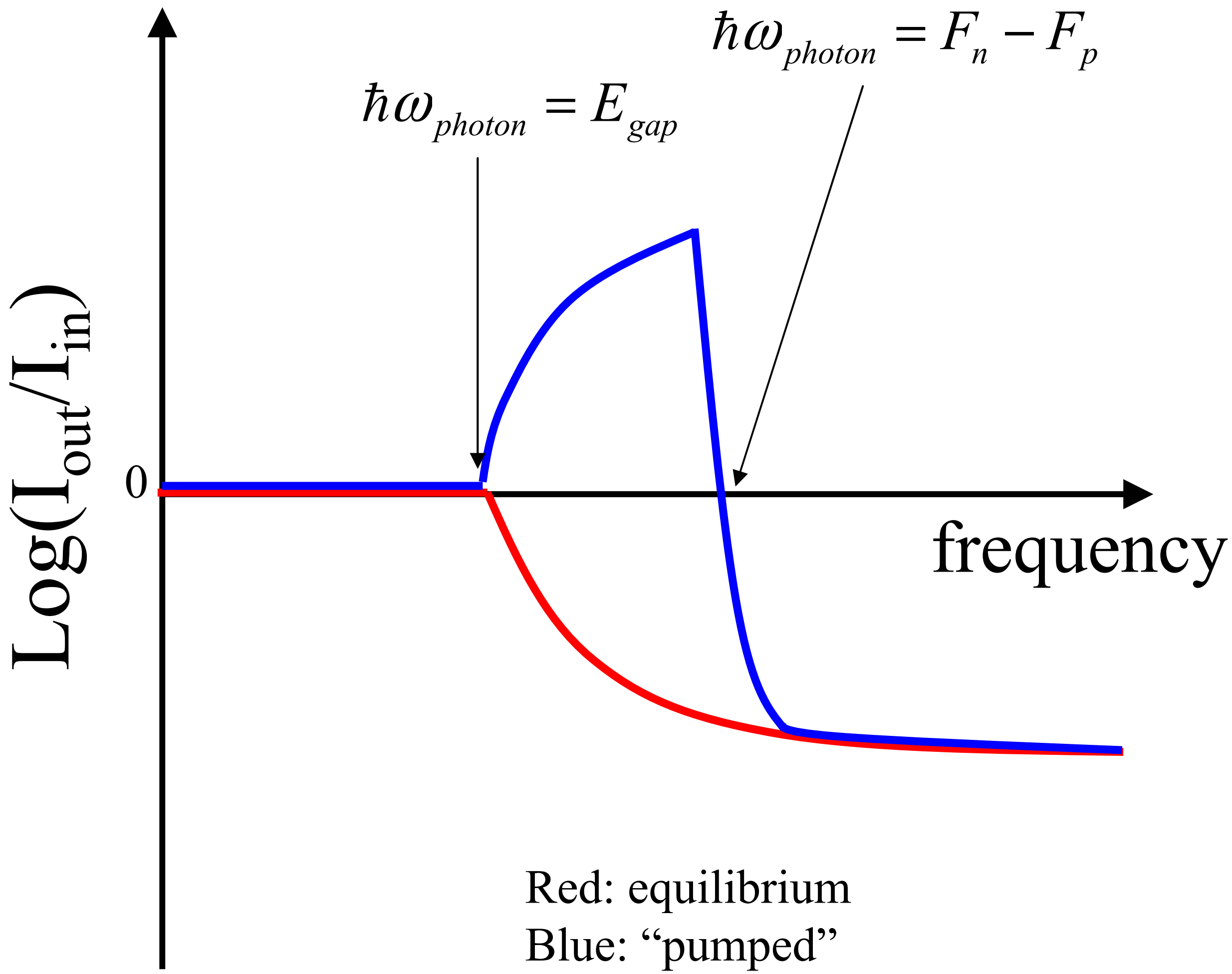


Discuss diagram in detail, including overlay of density of states, photon energy, and

Optical absorption (?):



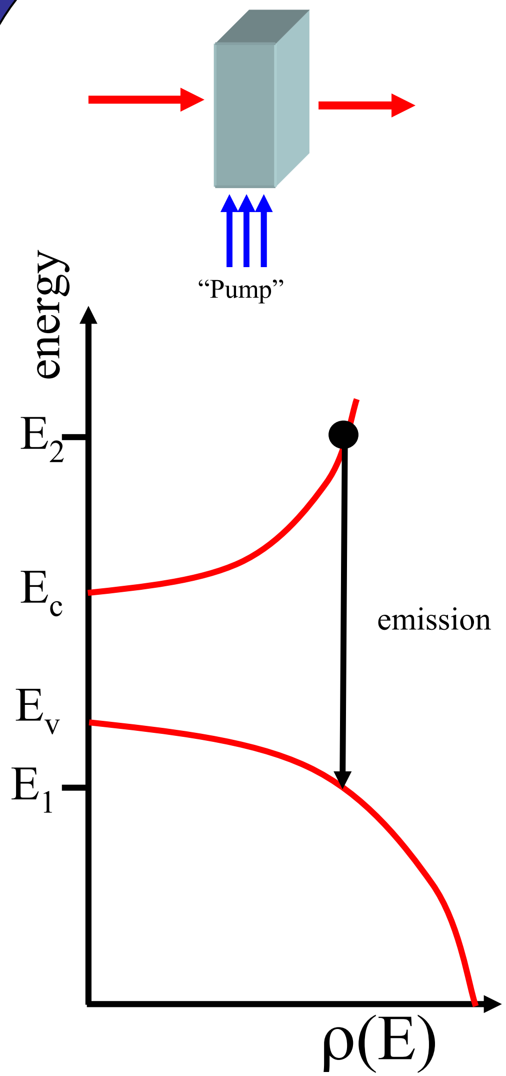




Discuss log scale on board (0).

Our goal in the next slides is to calculate quantitatively the blue curve.

Optical emission:



$$\hbar\omega_{photon} = E_2 - E_1$$

$$\hbar k_c = \sqrt{2m_e^*(E_2 - E_c)}$$

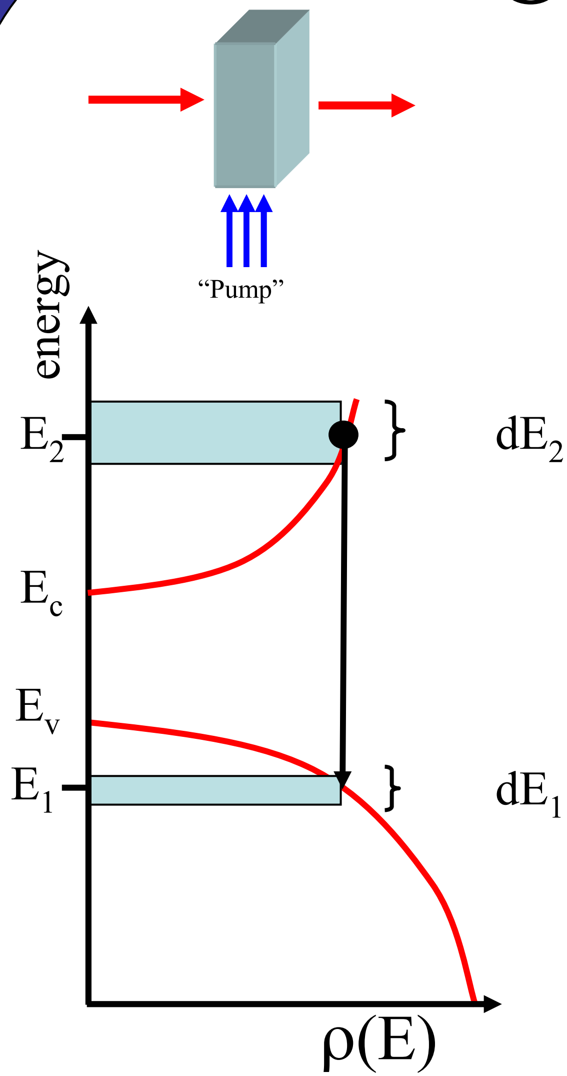
$$\hbar k_v = \sqrt{2m_h^*(E_v - E_1)}$$

$$\hbar k_v = \hbar k_c$$

$$\Rightarrow E_2 - E_c = \frac{m_h^*}{m_e^*}(E_v - E_1)$$

E_2 and E_1 are *not* centered around midgap.

Optical emission:

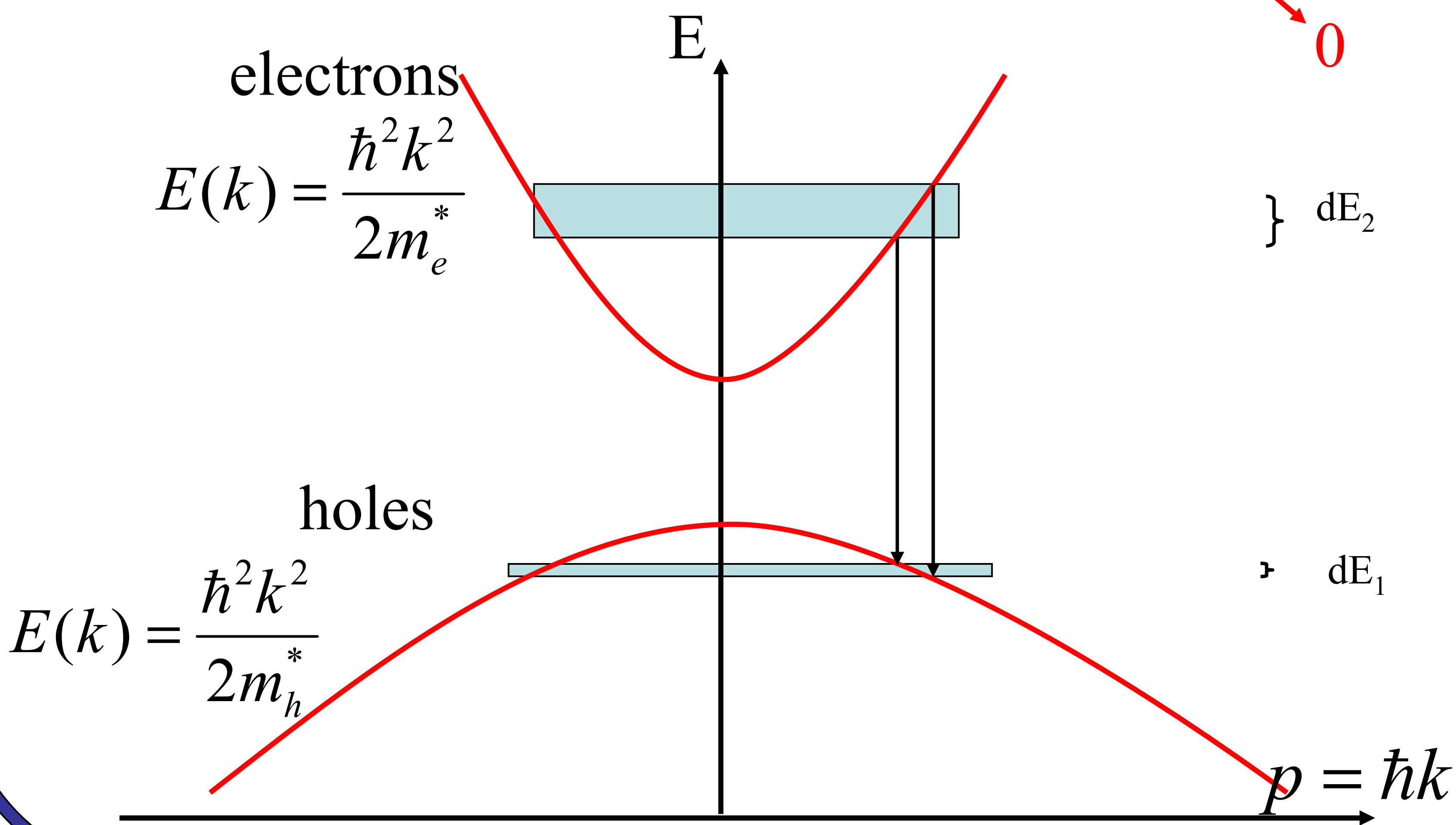


dE_1 and dE_2?

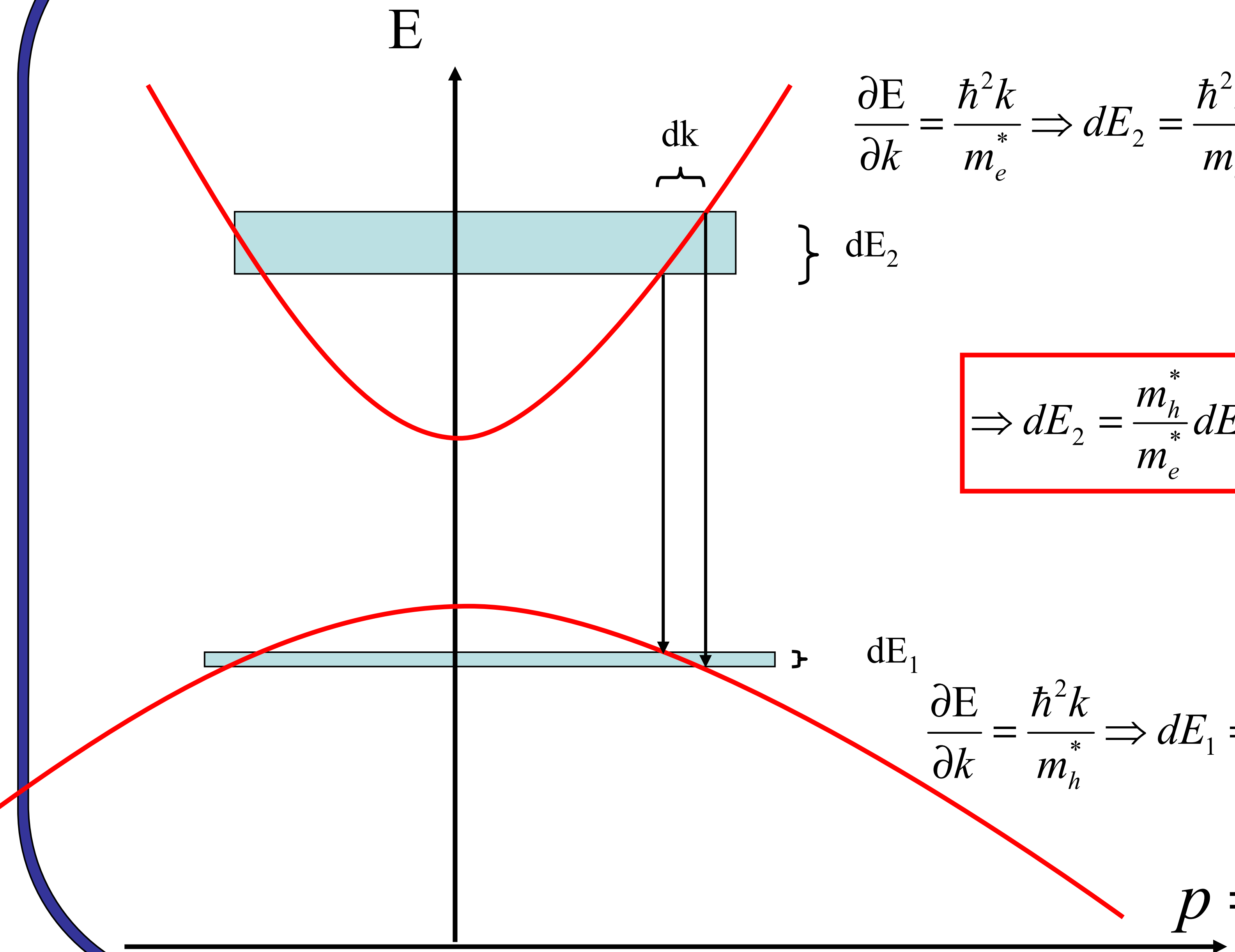
Optical emission:

$$E_{electron}(before) = E_{electron}(after) + \hbar\omega_{photon}$$

$$p_{electron}(before) = p_{electron}(after) + p_{photon}$$



Optical emission:

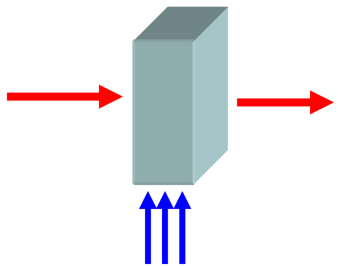


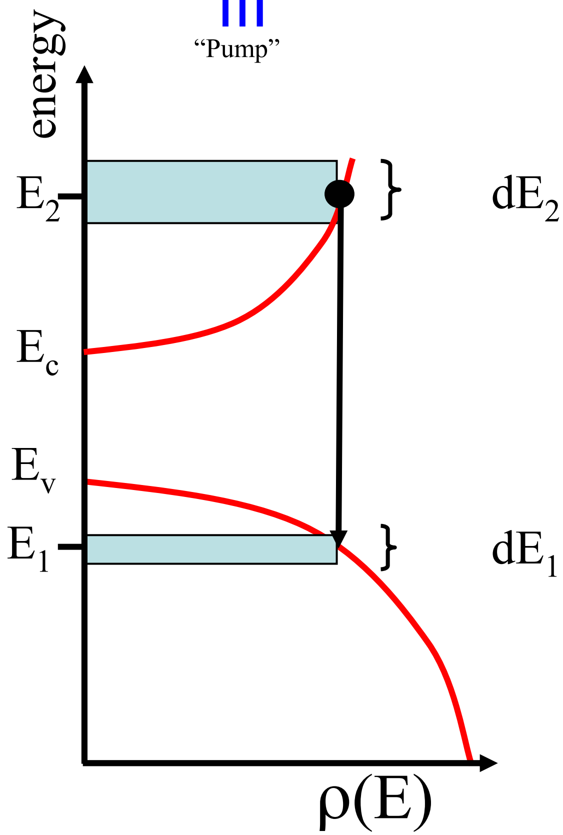
$$\frac{\partial E}{\partial k} = \frac{\hbar^2 k}{m_e^*} \Rightarrow dE_2 = \frac{\hbar^2 k}{m_e^*} dk$$

$$\Rightarrow dE_2 = \frac{m_h^*}{m_e^*} dE_1$$

$$\frac{\partial E}{\partial k} = \frac{\hbar^2 k}{m_h^*} \Rightarrow dE_1 = \frac{\hbar^2 k}{m_h^*} dk$$

So far:





$$E_2 - E_c = \frac{m_h^*}{m_e^*}(E_v - E_1)$$

$$dE_2 = \frac{m_h^*}{m_e^*} dE_1$$

So far, we have assumed:

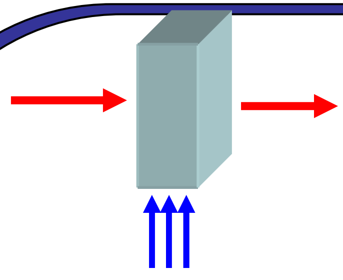
$$\hbar\omega_{photon} = E_2 - E_1$$

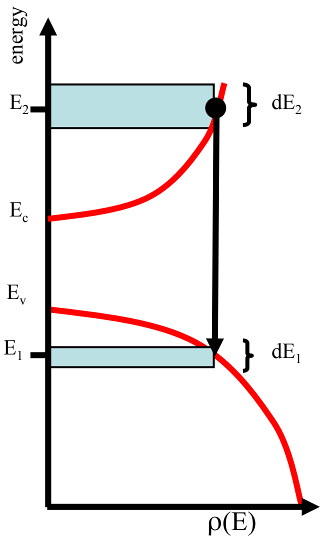
What if the incoming intensity is distributed in frequency such that I(ν)dν is the intensity between ν and ν+dν?

How many states are there such that an incident intensity with total intensity I(ν)dν participates?

$$\rho_{jnt}(h\nu) = \frac{1}{2}\left[\frac{1}{\rho_c(E_2)} - \frac{1}{\rho_v(E_1)}\right]$$

So far:





$$R_{1\to 2} = B_{1-2}\cdot\frac{1}{c}I(\upsilon)d\upsilon\cdot\rho_{jnt}(\nu)\cdot\left[f_v(E_1)\left(1-f_c(E_2)\right)\right]$$

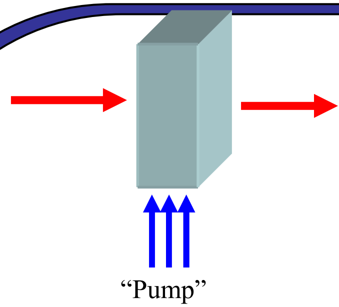
$$R_{2\to 1} = B_{2-1}\cdot\frac{1}{c}I(\upsilon)d\upsilon\cdot\rho_{jnt}(\nu)\cdot\left[f_c(E_2)\left(1-f_v(E_1)\right)\right]$$

$$R_{2\to 1} - R_{1\to 2} = B_{2-1}\cdot\frac{1}{c}I(\upsilon)d\upsilon\cdot\rho_{jnt}(\nu)\cdot\left[f_c(E_2)-f_v(E_1)\right]$$

Power emitted = hν * number of transitions/time = hν * (R_{21}-R_{12})

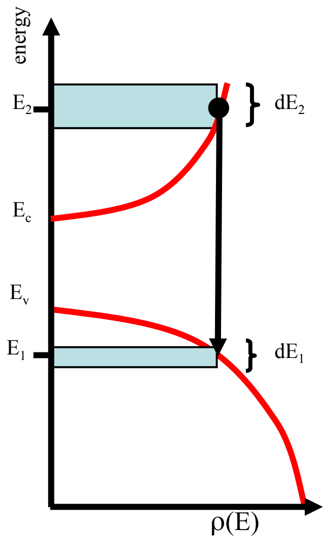
$$\gamma(\upsilon) \equiv \frac{dI(\upsilon)/dz}{I(\upsilon)} = \frac{power/volume}{I(\upsilon)} = \frac{h\upsilon\cdot\left[R_{2\to 1}-R_{1\to 2}\right]}{I(\upsilon)d\upsilon}$$

$$\gamma(\upsilon) = B_{2-1}\cdot\frac{1}{c}\cdot\rho_{jnt}(\nu)\cdot\left[f_c(E_2)-f_v(E_1)\right]$$



So far:

$$\gamma(\upsilon) = B_{2-1} \cdot \frac{1}{c} \cdot \rho_{jnt}(\nu) \cdot [f_c(E_2) - f_v(E_1)]$$

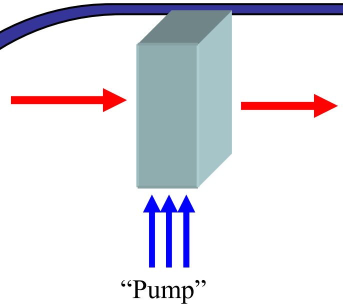


$$\rho_{jnt}(\nu) = \left(\frac{2m_e^* m_h^*}{m_e^* + m_h^*} \right)^{1/2} \sqrt{h\upsilon - E_{gap}}$$

$$\boxed{\gamma(\upsilon) = K \cdot \sqrt{h\upsilon - E_{gap}} \cdot [f_c(E_2) - f_v(E_1)]}$$

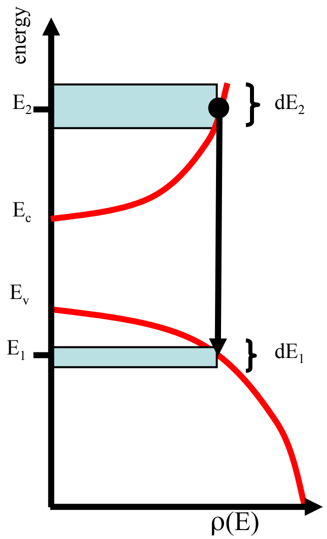
Three cases:

1) $h\nu < E_{gap}$
2) $E_{gap} < h\nu < F_n - F_p$
3) $h\nu > F_n - F_p$



Case one: hν < E_{gap}

$$\gamma(\upsilon) = K \cdot \sqrt{h\upsilon - E_{gap}} \cdot [f_c(E_2) - f_v(E_1)]$$

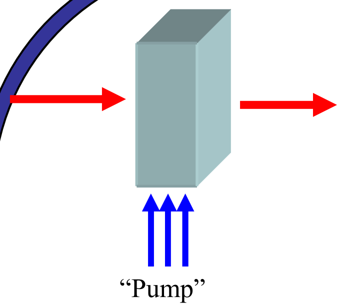


No absorption

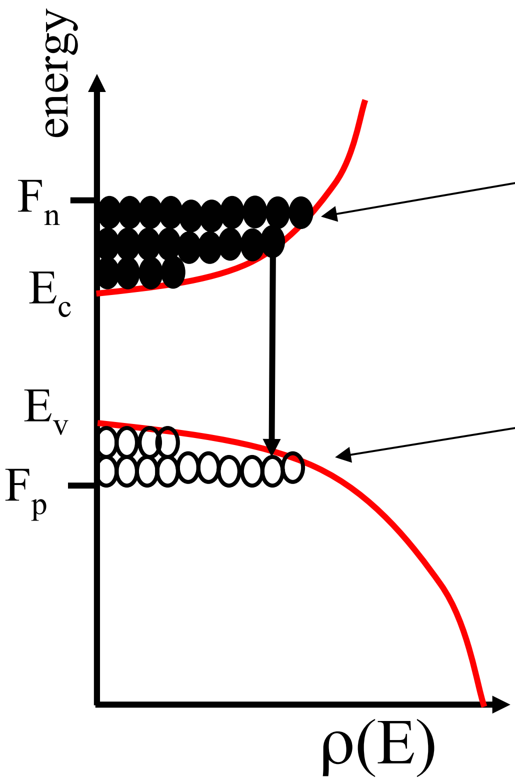
No emission

$\gamma = 0$

Case two: $E_{gap} < h\nu < F_n - F_p$



$$\gamma(\upsilon) = K \cdot \sqrt{h\upsilon - E_{gap}} \cdot \left[f_c(E_2) - f_v(E_1) \right]$$

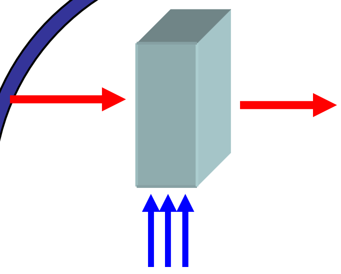


$$f_c(E_2) \approx 1$$

$$f_v(E_1) \approx 0$$

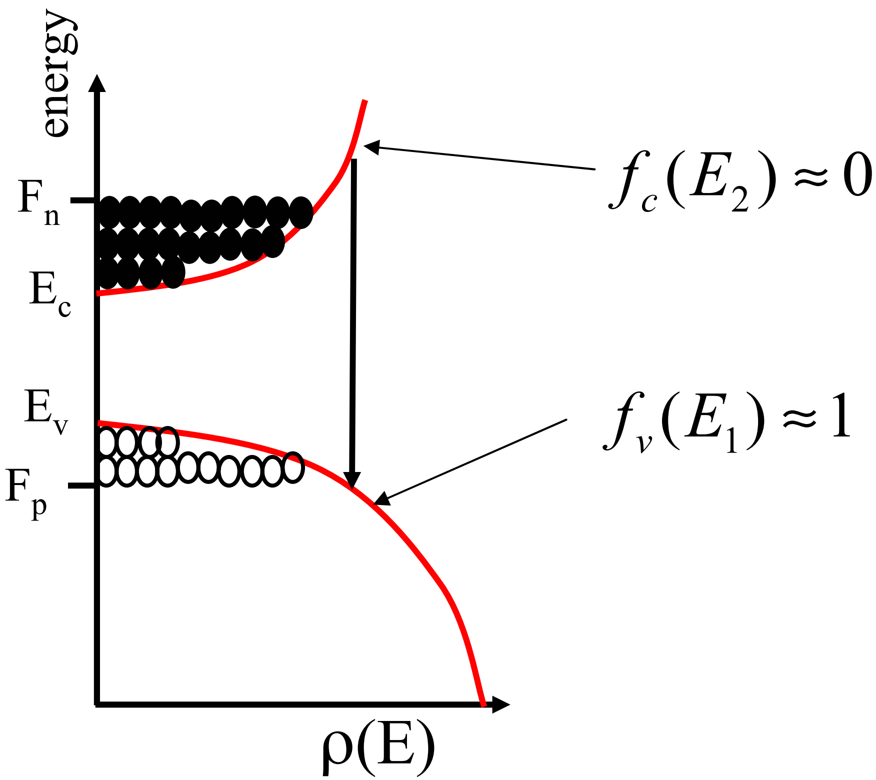
$$\gamma(\upsilon) \approx K \cdot \sqrt{h\upsilon - E_{gap}}$$

GAIN



Case three: > F_n-F_p

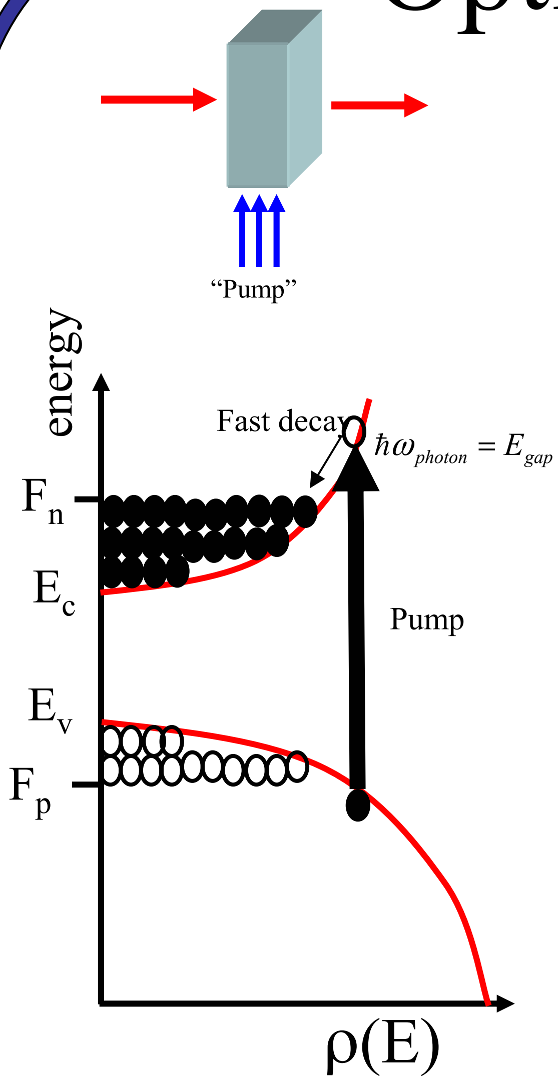
$$\gamma(\upsilon) = K \cdot \sqrt{h\upsilon - E_{gap}} \cdot \left[f_c(E_2) - f_v(E_1) \right]$$

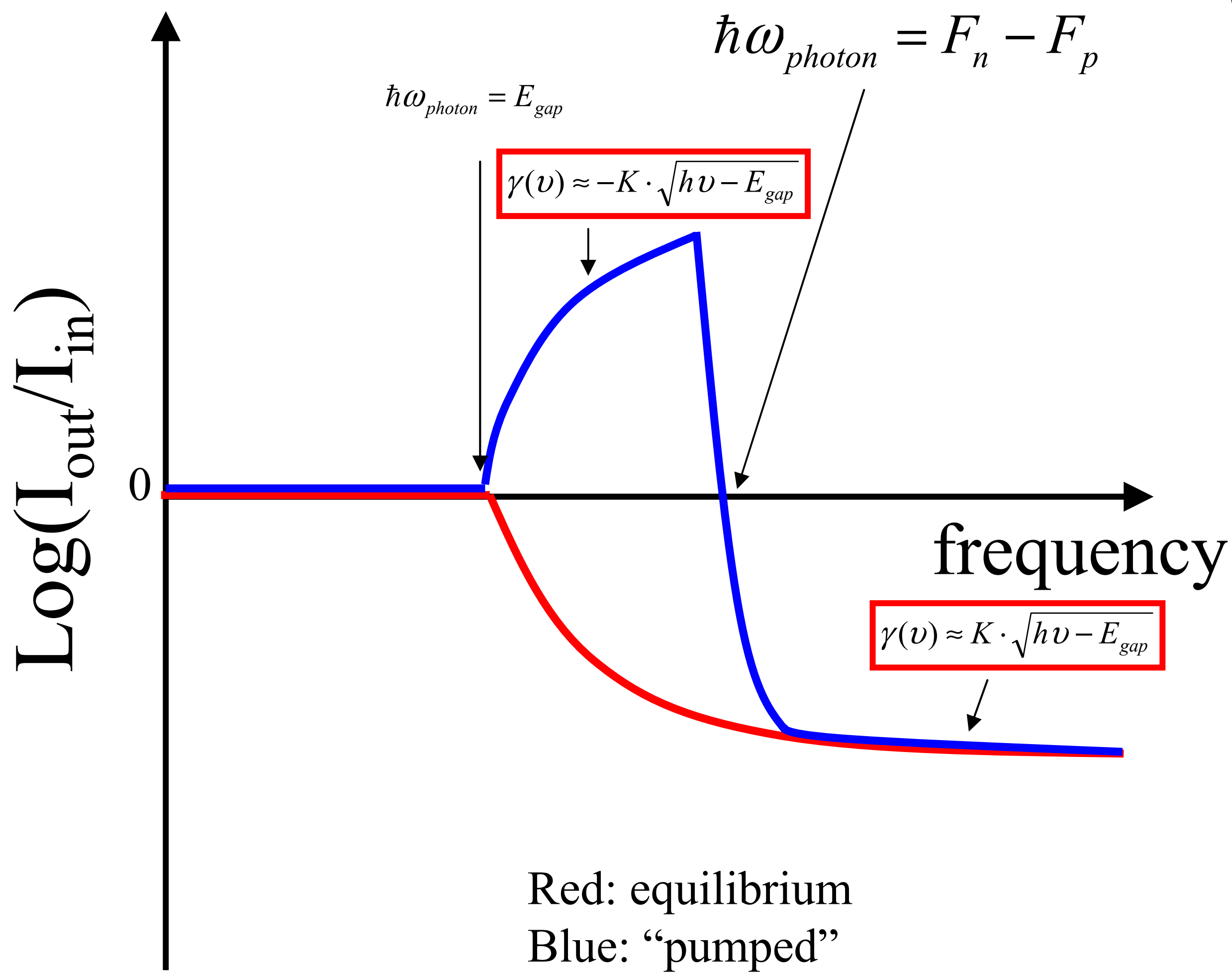


$$\gamma(\upsilon) \approx -K \cdot \sqrt{h\upsilon - E_{gap}}$$

LOSS

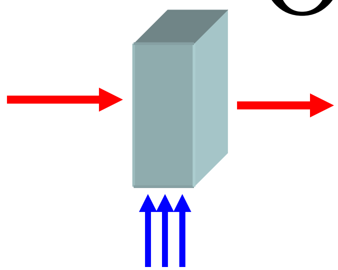
Optical absorption (?):

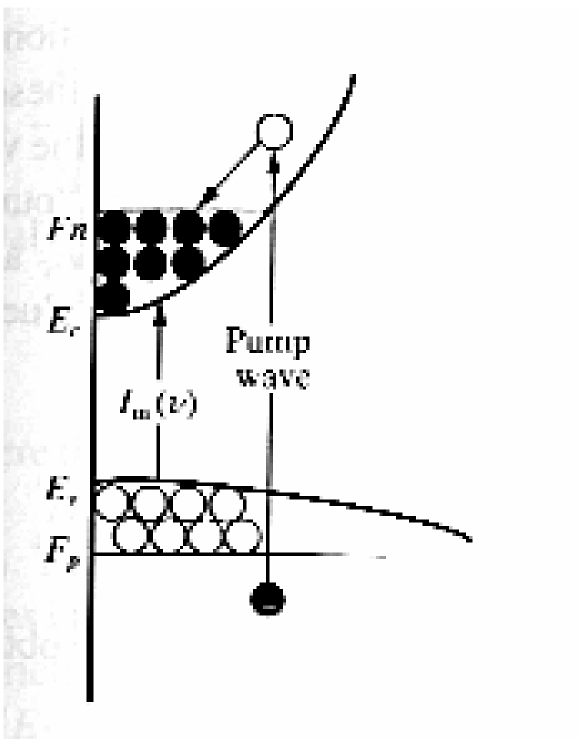




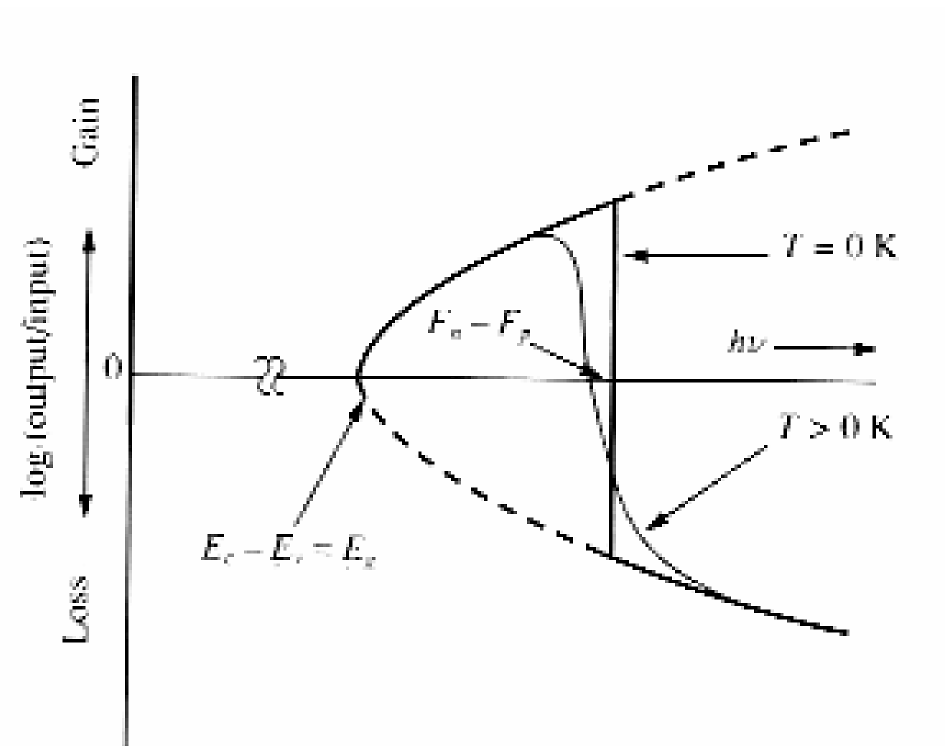
Our goal in the next slides is to calculate quantitatively the blue curve.

Optical absorption (?):





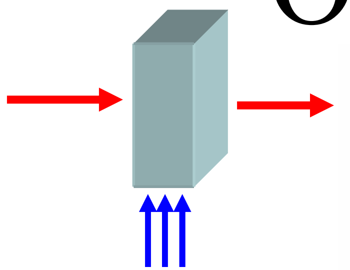
(b) The band diagram



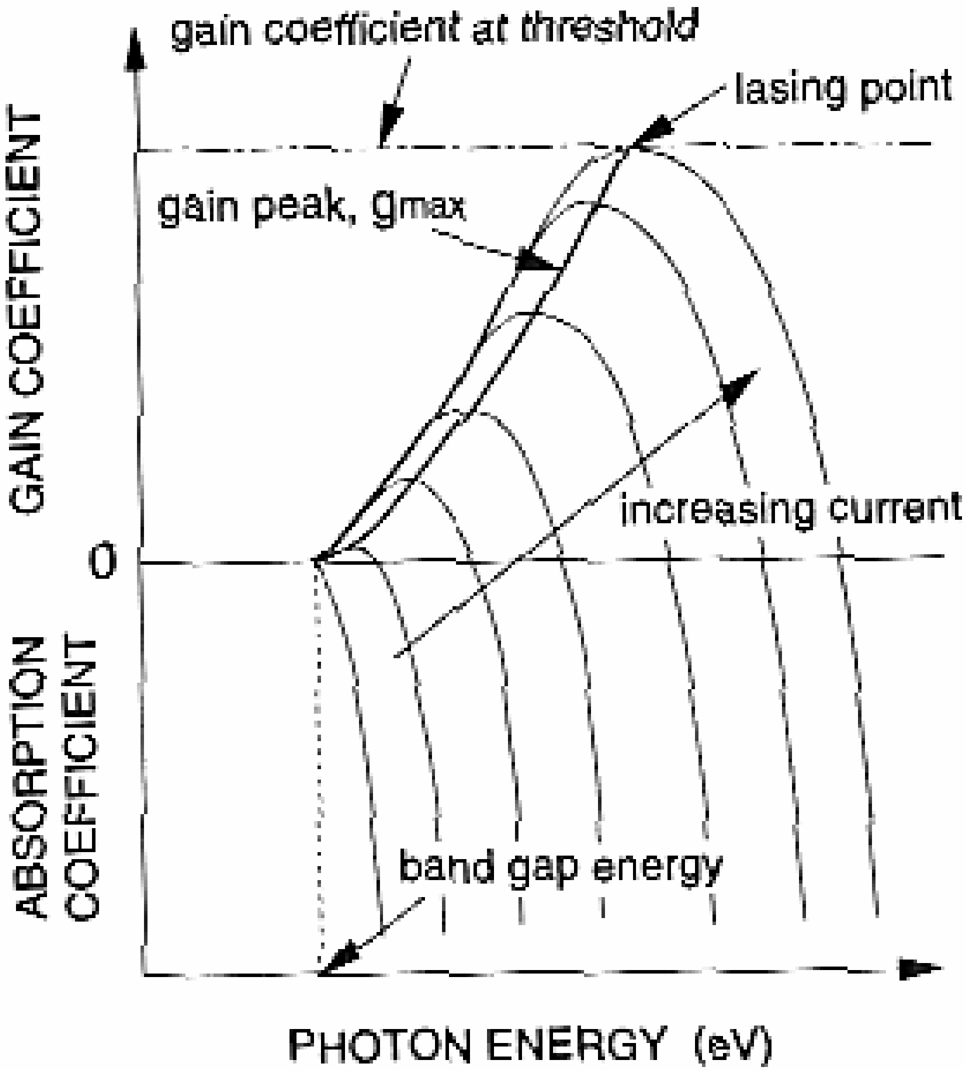
(c) The "data"

From Verdeyen

Optical absorption (?):



Discuss log scale on board (0).



From Fukuda, Optical Semiconductor Devices