

ECE 278
Homework #5
Due Thursday, Feb. 14, 2002, at the beginning of class

1) (100 pts.)

A) Consider the four level system (0,1,2,3). The ground state is not a reservoir, so that the number of atoms is not constant in time. There is no “R” pumping rate. For the spontaneous emission, consider only the following decay paths: 3-0, 3-2, 2-1, 1-0. We shine light on the atoms that consists of two frequencies. The first frequency is given by $h\nu=(E_3-E_0)$. The second frequency is given by $h\nu=(E_2-E_1)$. So, you must consider stimulated emission from the 2-1 state, absorption from the 1-2 state, stimulated emission from the 3-0 state, and absorption from the 0-3 state. Write down the rate equations.

B) In the steady state, calculate the population inversion of the 2-1 level pair as a function of both intensities. In the limit that intensity 2-1 goes to zero and the limit that intensity 3-0 goes to infinity, you should recover the result from your previous homework. (You may use Mathematica or any other program to aid your algebra. My solutions might be wrong.)

C) Now, consider your answer to the $I_{21}=0$ and $I_{30}=\text{infinity}$ limit.

In the fictional three level scheme we discussed in lecture, we introduced a “pumping rate” R_2 which was the rate at which we took atoms from state 0 and put them into state 2. We did not specify a mechanism by which we accomplished this.

One possible way to accomplish this is to shine light at the system at the 0-3 transition frequency and let atoms spontaneously decay into state 2 out of level 3.

Compare with the results of the three level system in steady state with $I_{21}=0$. In our new four level problem, is it possible to determine an “effective” R_2 (it is) and replace the four level problem with the (simpler) three level problem with a simple re-identification of R_2 with some function of the parameters of the four-level problem? What exactly would R_2 be equal to then?

D) Now, consider the case where $I_{21}=0$ and I_{30} is non-zero but not infinite either. Calculate N_2-N_1 .

Compare with the results of the three level system in steady state with $I_{21}=0$. Same question: In our new four level problem, is it possible to determine an “effective” R_2 and replace the four level problem with the (simpler) three level problem with a simple identification of R_2 with some function of the parameters of the four-level problem? What exactly would R_2 be equal to then?

E) Now, consider the case where I_{21} is not zero but not infinite and I_{30} is infinite. Calculate N_2-N_1 .

Compare with the results of the three level system in steady state with I_{21} not zero. Same question: In our new four level problem, it is possible to determine an “effective” R_2 and replace the four level problem with the (simpler) three level problem with a simple identification of R_2 with some function of the parameters of the four-level problem? What exactly would R_2 be equal to then?

F) Now, consider the case where I_{21} is not zero but not infinite and I_{30} is not zero but not infinite. Calculate N_2-N_1 . (I guess you already did this in part B).

Compare with the results of the three level system in steady state with I_{21} not zero. Same question: In our new four level problem, it is possible to determine an “effective” R_2 and replace the four level problem with the (simpler) three level problem with a simple identification of R_2 with some function of the parameters of the four-level problem? What exactly would R_2 be equal to then?

If, in any of these cases, it is not possible to find a suitable definition of R_2 so that the answers are similar to the three level problem, discuss reasons for why it might not be possible to do this.