ECE 278
Homework #2 (corrected version)
Due Jan. 22, 2002, at the beginning of class

1) (15 pts.) At a temperature of T=300 K, what is the frequency at which there is the maximum black-body radiation energy per frequency range dv? What is the wavelength at which there is the maximum blackbody energy per wavelength range dλ? (Be very careful. Something is easy to be confused.)
2) (20 pts.) Anybody who has done a thermal evaporation knows the following experimental fact: As you increase the temperature of the source, it begins to glow. First, it glows red, then eventually at a high enough temperature it glows white. Using the Planck distribution function, estimate the temperature at which the boat begins to glow red. Estimate the temperature at which the boat begins to glow white. Is a white-hot temperature high enough to melt gold? Is it high enough to melt aluminum? Is it high enough to melt Tungsten? (I hope the answer to the last question is no. This is how a light bulb works: you get the tungsten filament "white hot" without melting it.) Hint: white light has an intensity which is independent of frequency in the optical band.
3) (20 pts.) By integrating the spectral density of the Planck intensity vs. frequency from zero to infinity, derive the Stephan-Boltzman law:
   $P_{emitted} = \sigma A T^4$
   Calculate the constant of proportionality called σ. Next, calculate roughly how much power a human being emits, and compare that to the amount of power emitted by an object the same size as a human being, but that is at T=0 C. (Aside: If you can build a detector that can measure the difference, then you can see people in the dark.)
4) (20 pts.) The gain bandwidth of various lasers was discussed in class. From the table in the notes, roughly what length does the cavity need to be for each type of laser in order to operate single mode? (Make another table).
5) (10 pts.) Prove the statement in class about standing waves: (Assume $E_0 = E_{0real} + i\, E_{0imag}$, and $E_{0real} = 0$.)

$$\vec{E}(\vec{r},t) = \mathrm{Re}\left[E_0\, \hat{z}\, e^{i(k\hat{x}\cdot\vec{r} - \omega t)} \right] + \mathrm{Re}\left[E_0\, \hat{z}\, e^{i(k\hat{x}\cdot\vec{r} + \omega t)} \right] = -2E_{0imag}\, \hat{z} \sin(k_n x)\cos(\omega t)$$

6) (15 pts.) Prove that $\dfrac{\sum_{n=0}^{\infty} nh\nu \cdot e^{-nh\nu/k_B T}}{\sum_{n=0}^{\infty} e^{-nh\nu/k_B T}} = \dfrac{h\upsilon}{e^{h\nu/k_B T} - 1}$ You may use whatever method you like, but I have

   some hints if you are interested. First, work on the denominator. Use that fact that $\sum_{n=0}^{\infty} x^n = 1/(1-x)$ and write $e^{-nh\nu/k_B T}$ as $\left(e^{-h\nu/k_B T}\right)^n$. Next, work on the numerator. Express the numerator as $\sum_{n=0}^{\infty} n \cdot a \cdot e^{-n\cdot a}$. (You will need to pull out a factor of kT.) Next consider the following:

$$\sum_{n=0}^{\infty} n \cdot a \cdot e^{-n\cdot a} = \left[\frac{d}{dy} \sum_{n=0}^{\infty} e^{-n\cdot a\cdot y} \right]_{y=1} = \left[\frac{d}{dy} \sum_{n=0}^{\infty} \left(e^{-a\cdot y}\right)^n \right]_{y=1} = \left[\frac{d}{dy} \left(\frac{1}{1-e^{-a\cdot y}} \right) \right]_{y=1}$$

   . Continue by evaluating the last derivative of y, and you will be able to remove the summation formula from the numerator. Combine the numerator and the denominator to finish the proof.