NAME______________________

Student ID #______________________

| 1 | 2A | 2B | 2C | 3A | 3B | Total |
|---|---|---|---|---|---|---|
| /20 | /10 | /20 | /20 | /15 | /15 | /100 |

1) (20 points) Consider a Si based laser. Assume that the Si is the gain medium. (In reality it is hard to get gain out of Si, but ignore that for the moment.) The mirror is the cleaved facet. The relative dielectric constant for Si is 11.9. From that you should be able to figure out the index of refraction (=square root of relative dielectric constant).

If the only loss is due to the finite reflectivity of the end of the laser, what does the (small intensity) one-way gain of the "gain medium" need to be to sustain lasing? The ends reflect because of the index of refraction mismatch between the Si and vacuum. Express your answer in dB.

Si laser
cleaved facet

**Lecture #1 Slide # 34:**

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \left(\frac{\sqrt{11.9} - 1}{\sqrt{11.9} + 1}\right)^2 = \left(\frac{2.45}{4.45}\right)^2 = (0.55)^2 = 0.303$$

**This means that 30.3% of the power incident on the "mirror" (i.e. edge of device) is reflected.**
**This corresponds to a power loss of -10 log (0.303) = 5.1 dB.**
**So the gain must be + 5.1 dB to achieve lasing.**

**Grading criteria: Correct answer for correct reason, 20 pts.**
**Didn't square reflection law, else correct, 10 pts.**
**Didn't use reflection law, but instead 1/n, 5 pts.**

2) (50 points) Imagine a laser made of a gain medium with the following parameters:
L=1 m (distance between mirrors), gain bandwidth=$10^{11}$ Hz, center frequency = 650 nm:

A) Calculate the free spectral range. (10 points)

**Lecture #8, slide #6:**

$$\Delta\omega = \frac{c\pi}{L} = 9.42x10^8 Rad/s$$

$$\Delta f = \frac{c}{2L} = 1.5x10^8 \frac{1}{s} = 1.5x10^8 Hz = 150MHz = 0.15GHz$$

**Note: 9.42 x $10^8$ Hz is not correct.**
**Units are as follows:**

$$Hz \equiv \frac{1}{s}$$

$$Rad/s \neq Hz$$

**Grading criteria: Correct answer 10 pts.**
**Calculate FSR in wavelength units only, 5 pts.**
**Get Hz right but Rad/s wrong 9 pts.**
**Right formula, wrong answer 5 pts.**
**Wrong formula, wrong answer: 0 pts.**

B) If the broadening is inhomogenous, decide whether the laser will operate in single or multi mode. If you answer is multimode, estimate how many modes will lase. (20 points)

**# modes = Gain BW/ FSR = $10^{11}$ Hz/1.5x$10^8$ Hz=666.**
**Incorrect formula, 0 pts.**
**Correct eqn, wrong answer because of part A, 10 pts.**
**Correct eqn., wrong answer because of arithmetic, 10 pts.**

C) If the broadening is homogeous, decide whether the laser will operate in single or multi mode. If you answer is multimode, estimate how many modes will lase. (20 points)

**Lecture #8 Slide #31:**
**Single mode, because homogenous (20 pts.)**
**Say single mode but for wrong reason, 10 pts.**

3) (30 points) An experiment involving a homogeneously broadened optical amplifier is performed. For an input intensity of 1 W/cm2, the gain (output/intput) is 10 dB. For an input intensity of 2 W/cm2, the gain is reduced to 9 dB.

A) What is the small signal gain (i.e. $I_{in}$ -> 0) of this amplifier (in dB)? (15 points)

**See attached.**
**Grading criteria (both parts):**
**Correct answer, 15 pts.**
**Correct approach, arithmetic error, 10 pts.**
**Correct approach, incorrect use of gain (linear/log) during implementation, 9 pts.**
**Correct approach, but don't understand small signal gain is e^($\gamma_0 l_g$), 5 pts.**
**Didn't write or use complicated $I_{out}$ vs $I_{in}$ eqn., 0 pts.**
**Write complicated $I_{out}$ vs $I_{in}$ eqn. but everything else wrong, 1 pts.**

B) What is the saturation intensity? (15 points)

**See attached.**

# Chapter 8

8.1

$$\sigma = A_{21}\frac{\lambda_0^2}{8\pi}\left[g(\nu_0) = \frac{2}{\pi\Delta\nu_h}\right] = 9.68\times10^{-18}\ \text{cm}^2;\ \gamma_0 = 0.05\ \text{cm}^{-1} = \sigma\cdot\Delta N;$$

$\Delta N = 5.17 \times 10^{+15}\ \text{cm}^{-3}$; $I_s = h\nu/\sigma\tau_2 = 193.5\ \text{W/cm}^2$

8.2

(a) $\ln\frac{I_{out}}{I_{in}} + \frac{I_1}{I_s}\left(\frac{I_{out}}{I_{in}} - 1\right) = \gamma_0 l_g;$

For an input $I_1 = 1\text{W/cm}^2$, $\quad \ln(10) + \frac{I_1}{I_s}(10-1) = \gamma_0 l_g;$ 1.

For the input $= 2\text{W/cm}^2 = 2\,I_1$: $\quad \ln 8 + 2\frac{I_1}{I_s}(8-1) = \gamma_0 l_g;$ 2.

Subtract the two equations: $\quad \ln(10/8) = (I_1/I_s)\,(14-9);$ 3.

Thus: $I_1/I_s = 0.0446$ and (b) $\quad I_s = 1/0.0446 = 22.4$ watts/cm$^2$ 4.

Substitute this result back into 1. $\gamma_0 l_g = \ln(10) + 0.0446 \times 9 = 2.704;$

Hence $\quad G_0 = e^{\gamma_0 l_g} = 14.94$ or 11.74 dB 5.

(c) Maximum extractable intensity $= (\gamma_0 l_g)\cdot I_s = 2.704 \times 22.4 = 60.6\ \text{W/cm}^2$

(d) If $I_{out} - I_{in} = 0.5\,[\gamma_0 l_g \cdot I_s] = 30.3\ \text{W/cm}^2$ with $\ln\frac{I_0}{I_{in}} + \frac{I_0 - I_{in}}{I_s} = \gamma_0 l_g$,

Thus, we have: $\quad \ln\frac{I_0}{I_{in}} + \frac{1}{2}\gamma_0 l_g = \gamma_0 l_g$ or $\ln\frac{I_0}{I_{in}} = \frac{2.704}{2}$ or $\frac{I_0}{I_{in}} = 3.866$

$I_{in}\,(3.866 - 1) = 30.3 \quad \therefore I_{in} = 10.6\ \text{W/cm}^2$

8.3

$$\frac{df}{dz} = \left(\frac{\gamma_0}{1+f} - \alpha\right)f, \text{ where } f = I/I_s \qquad 1.$$

Integrate: $$\int_{f_1}^{f_2} \frac{(1+f)\,df}{f\{1 - [\alpha/(\gamma_0-\alpha)]\,f\}} = (\gamma_0-\alpha)\int_0^{l_g} dz$$

$$\ln\left[\frac{f_2}{f_1}\right] - \left[\frac{\gamma_0}{\alpha}\right]\ln\left\{\frac{(\gamma_0/\alpha - 1) - f_2}{(\gamma_0/\alpha - 1) - f_1}\right\} = (\gamma_0 - \alpha)\,l_g \qquad 2.$$

where $f_{1,2}$ = Intensity at (input,ouput)

Let $G_0 = \exp[(\gamma_0-\alpha)\,l_g]$, the net small signal gain ; $m = \gamma_0/\alpha$; $G_s = \frac{I_2}{I_1}$ (saturated gain);

Then the above becomes:

$$\ln G_s + \ln\left\{\frac{(m-1) - x}{(m-1) - G_s x}\right\}^m = \ln G_0 \text{ where } x = \frac{I_{in}}{I_s} \qquad 3.$$

For a given ratio $\gamma_0/\alpha$, one can solve this for $G_s$ in terms of x or one can solve for the normalized input, x, in terms of the ratio $(G_0/G_s)$. The latter is easier.