

# Lecture 9: Laser output power

- Reading: Verdeyen ch. 9
- Optional: Siegman ch. 12

## Steady-state intensity:

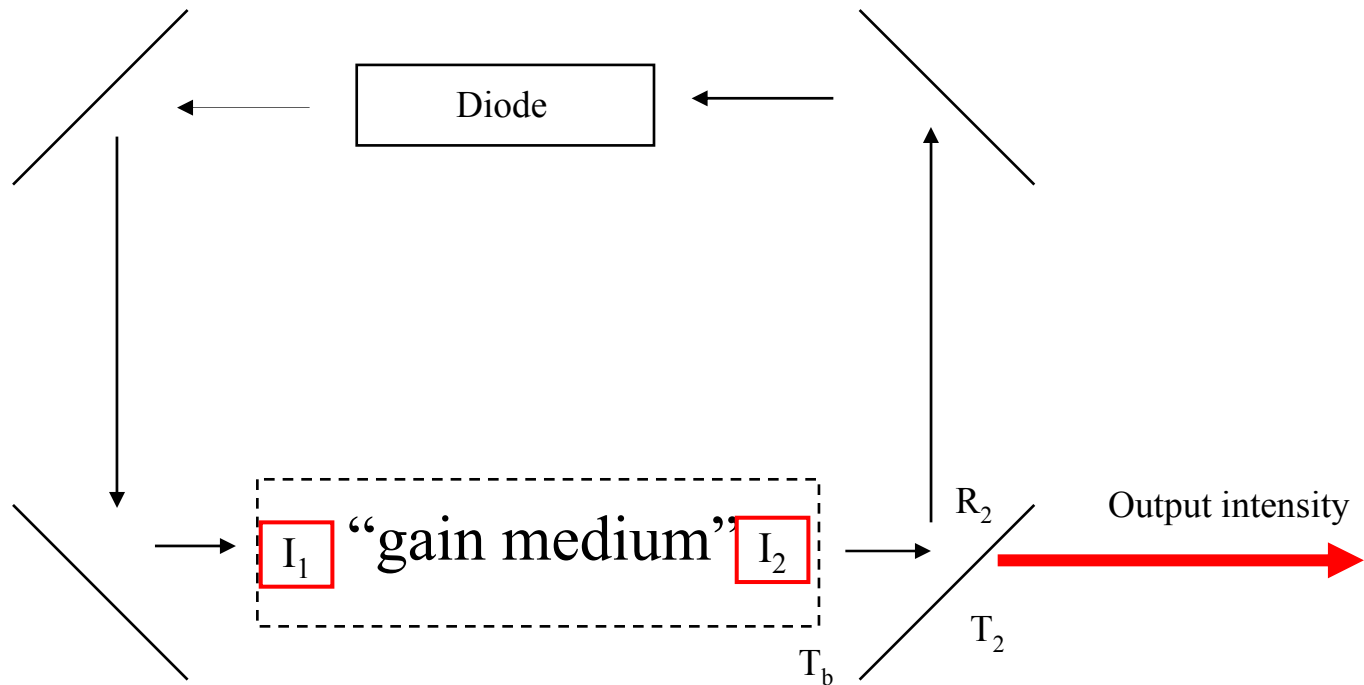
- Assume that we are not letting light out yet.
- There is only absorption in the cavity.
- What is intensity in the cavity?
- Gain is saturated at loss (otherwise intensity would increase with time)
- Loss is a property of the cavity
- Gain is function of intensity
- Find intensity that gives the gain=loss, and that is the internal intensity of cavity.
- $I_s$  (function of lifetime and frequency) important scale of intensity to expect.

# Output power

- Very reflective mirrors: Internal intensity high, but little of it gets out
- Very transmitting mirrors: Lots of the light gets out but internal intensity is low.
- Which is better? Need a quantitative theory.
- Ring laser
- Standing wave laser

# Ring laser

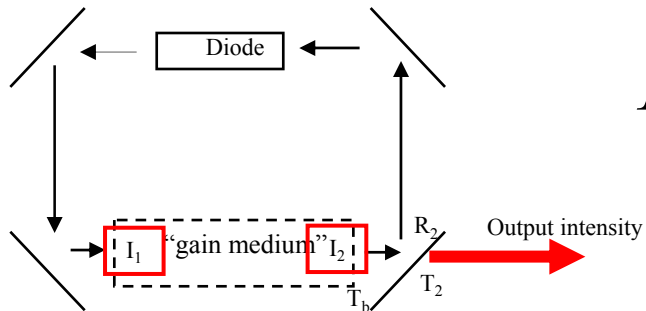
Easy to analyze because light only propagates in one direction.



$$I_2 \rightarrow I_2 T_b \rightarrow I_2 T_b R_2 \rightarrow I_2 T_b R_2 L = I_1 \rightarrow G I_1 = G(I_2 T_b R_2 L) = I_2$$

But we have a formula relating  $I_2$  and  $I_1$ .

# Ring laser



$$I_2 T_b R_2 L = I_1 \Rightarrow I_2 / I_1 = T_b R_2 L$$

$$\ln \frac{I_2}{I_1} + \frac{I_2 - I_1}{I_s} = \gamma_0 l_g$$

$$\Rightarrow \frac{I_2}{I_s} = \frac{\gamma_0 l_g - \ln(T_b R_2 L)}{1 - T_b R_2 L}$$

Threshold: round trip loss \* gain = 1:

$$\Rightarrow T_b R_2 L \cdot e^{\gamma_{th} l_g} = 1 \Rightarrow \gamma_{th} = \frac{\ln(T_b R_2 L)}{l_g}$$

$$\Rightarrow \frac{I_2}{I_s} = \frac{\gamma_0 l_g - \gamma_{th} l_g}{1 - T_b R_2 L}$$

$$\Rightarrow I_{out} = I_2 \cdot T_b \cdot T_2 = T_b \cdot T_2 \cdot I_s \frac{\gamma_0 l_g - \gamma_{th} l_g}{1 - T_b R_2 L}$$

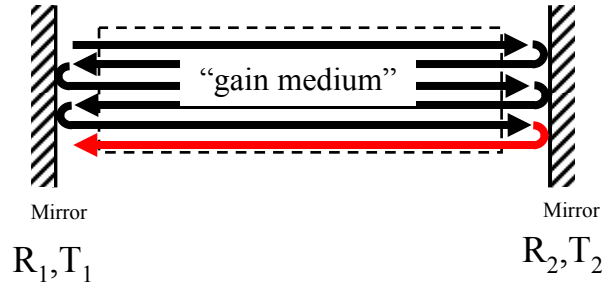
So we now know the output power!

## Pause and reflect:

- Started with plane waves
- Add mirrors
- Stimulated emission/absorption  $\rightarrow$  gain
- Optical reflectivities, losses, etc.
- All we need are lifetimes and energies, and we can predict the laser output power!
- Is it true? (I hope so!)

- Verdeyen goes on to discuss limits of highly reflective mirrors and highly transmitting mirrors for ring lasers.
- We will skip and go right to standing wave lasers.

# Standing wave laser:



We have waves going in both directions.  
The intensity of each is  $I_+(z)$  and  $I_-(z)$

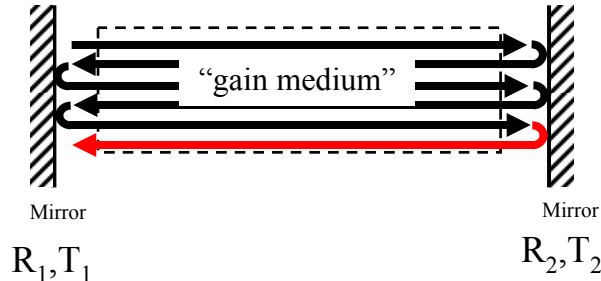
$\alpha_{\text{int}}, \alpha_{\text{ext}}$ : loss per unit length  
(discuss on board, put in to be consistent with Verdeyen)  
 $R_s, T_s$  a little different from Verdeyen. Forget  $\alpha_{\text{ext}}$

$$\frac{\partial I_+}{\partial z} = \gamma(z) I_+ \qquad \frac{\partial I_-}{\partial z} = -\gamma(z) I_-$$

$$\gamma = \frac{\gamma_0}{1 + I/I_s} \rightarrow \gamma(z) = \frac{\gamma_0}{1 + I(z)/I_s} = \frac{\gamma_0}{1 + (I_+(z) + I_-(z))/I_s}$$

Must solve for I!

# High Q approximation



“High Q approximation:”

$$R_1 \approx 1 \quad R_2 \approx 1$$

$\alpha_{\text{int}}$ : loss per unit length

Then intensity builds up to a high value internally.

Gain medium goes into saturation.

So we approximate  $I_+(z), I_-(z)$  as constant:  $\equiv I_{\text{circ}}$

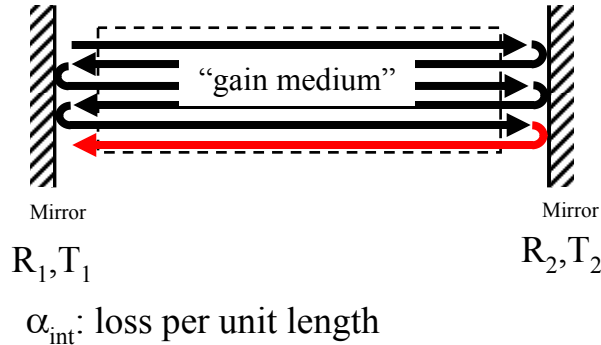
$$\gamma(z) = \frac{\gamma_0}{1 + (I_+(z) + I_-(z)) / I_s} = \frac{\gamma_0}{1 + (I_{\text{circ}} + I_{\text{circ}}) / I_s} = \frac{\gamma_0}{1 + 2I_{\text{circ}} / I_s}$$

$$\frac{\partial I_+}{\partial z} = \gamma(z) I_+ = \frac{\gamma_0}{1 + 2I_{\text{circ}} / I_s} I_+$$

$$\Rightarrow \text{One way gain} = e^{\frac{\gamma_0}{1 + 2I_{\text{circ}} / I_s} l_g}$$

(Not entirely self-consistent.)

# Standing wave laser (high Q):



$$\text{One way gain} = e^{\frac{\gamma_0}{1+2I_{\text{circ}}/I_s} l_g}$$

$$\text{Two way gain} = e^{\frac{2\gamma_0}{1+2I_{\text{circ}}/I_s} l_g}$$

$$\text{Two way gain w/loss} = e^{\frac{2\gamma_0}{1+2I_{\text{circ}}/I_s} l_g - 2\alpha_{\text{int}} l_g}$$

(Discuss on board)

Loss and gain must be equal for steady state:

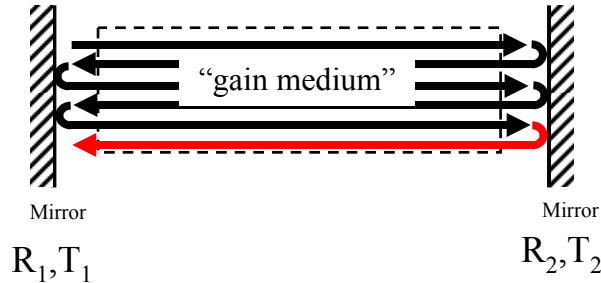
$$e^{\frac{2\gamma_0}{1+2I_{\text{circ}}/I_s} l_g} e^{-2\alpha_{\text{int}} l_g} = R_1 R_2 \quad \ln(xy) = \ln(x) + \ln(y)$$

$$\frac{2\gamma_0}{1+2I_{\text{circ}}/I_s} l_g - 2\alpha_{\text{int}} l_g = \ln(R_1) + \ln(R_2)$$

$$I_{\text{circ}} = \frac{1}{2} I_s \left[ \frac{\gamma_0 l_g}{\alpha_{\text{int}} l_g + \ln(1/R_2) + \ln(1/R_1)} - 1 \right]$$

# Standing wave laser

(high Q):



$\alpha_{\text{int}}, \alpha_{\text{ext}}$ : loss per unit length

$$I_{\text{circ}} = \frac{1}{2} I_s \left[ \frac{\gamma_0 l_g}{\alpha_{\text{int}} l_g + T_1 + T_2} - 1 \right]$$

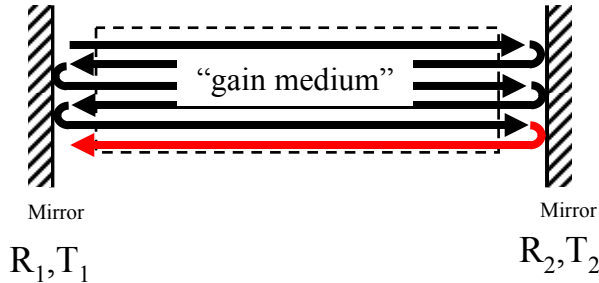
$$I_{\text{output}} = T_1 \cdot I_{\text{circ}} = T_1 \cdot \frac{1}{2} I_s \left[ \frac{\gamma_0 l_g}{\alpha_{\text{int}} l_g + T_1 + T_2} - 1 \right]$$

Simplify:  $R_2=1, T_2=0$

$$I_{\text{output}} = T_1 \cdot I_{\text{circ}} = T_1 \cdot \frac{1}{2} I_s \left[ \frac{\gamma_0 l_g}{\alpha_{\text{int}} l_g + T_1} - 1 \right]$$

Now let's ask: What value of  $R_1$  gives the largest  $I_{\text{output}}$ ?

# Output power (high Q):

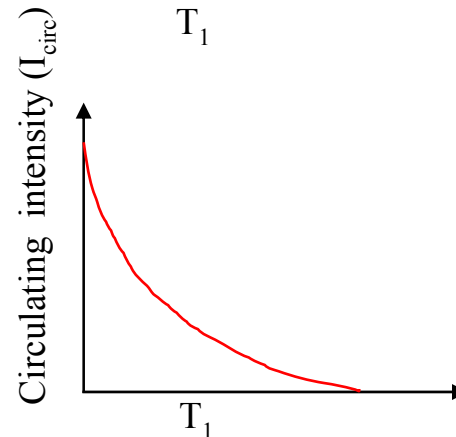
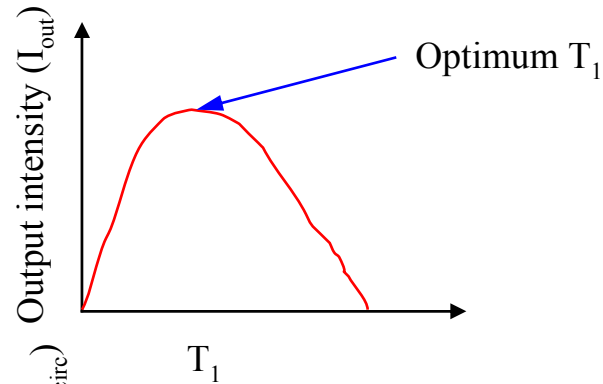


$\alpha_{\text{int}}, \alpha_{\text{ext}}$ : loss per unit length

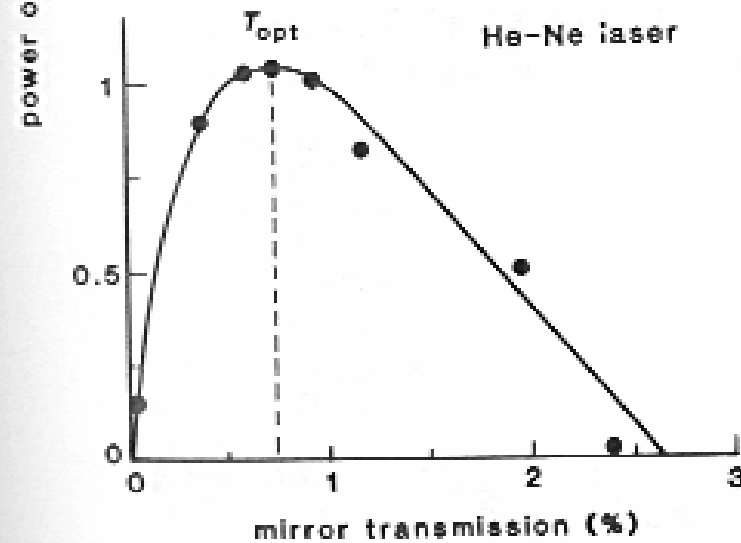
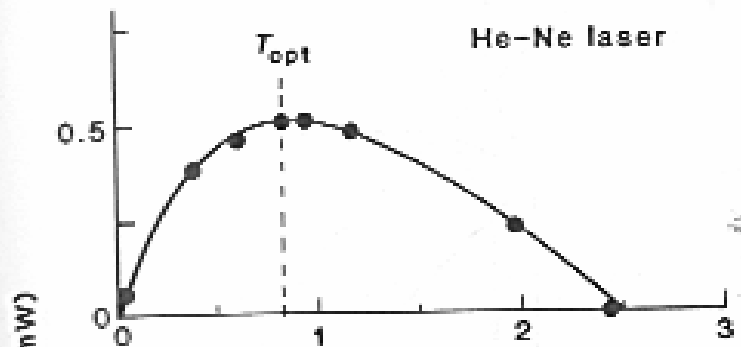
Now let's ask: What value of  $R_1$  gives the largest  $I_{\text{output}}$ ?

$$I_{\text{output}} = T_1 \cdot \frac{1}{2} I_s \left[ \frac{\gamma_0 l_g}{\alpha_{\text{int}} l_g + T_1} - 1 \right]$$

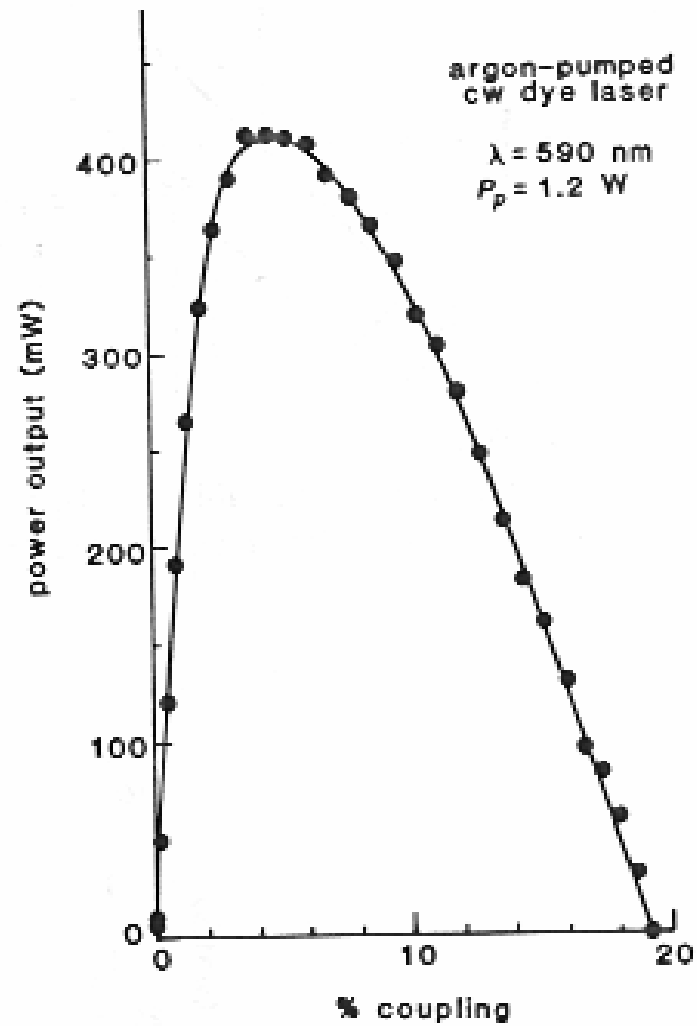
$$I_{\text{circ}} = \frac{1}{2} I_s \left[ \frac{\gamma_0 l_g}{\alpha_{\text{int}} l_g + T_1} - 1 \right]$$



# Is it true?



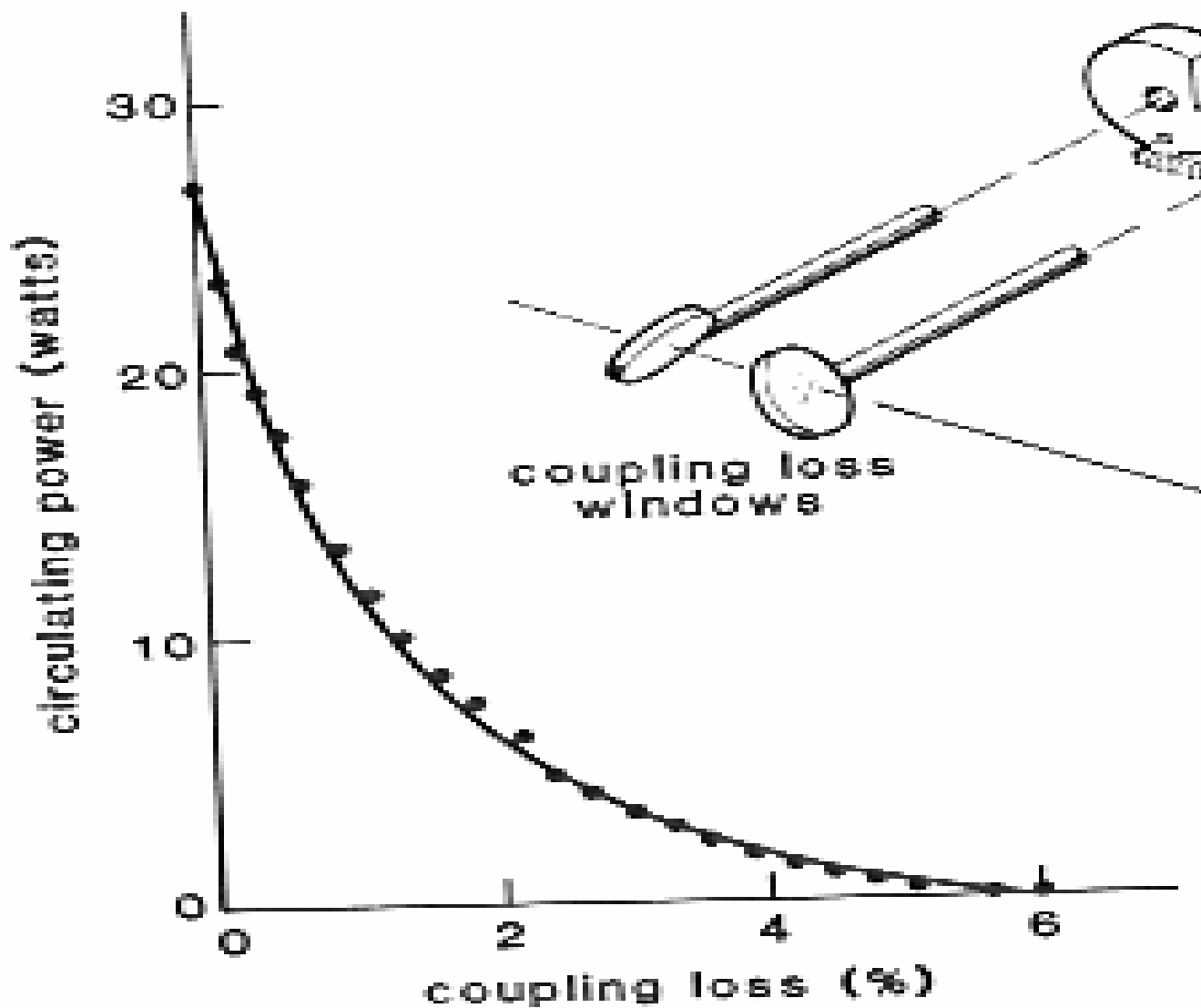
(a)



(b)

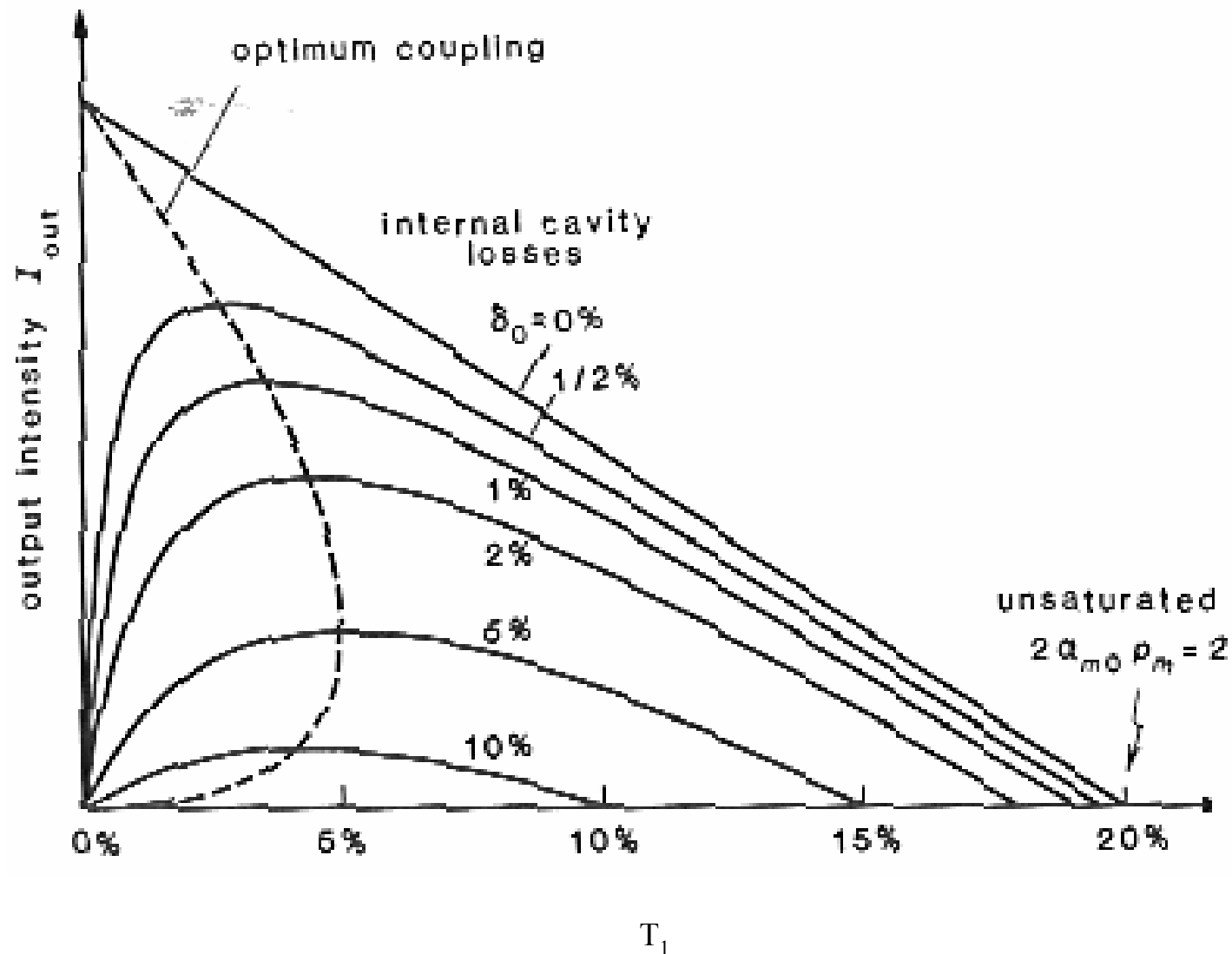
From Siegman, Lasers

# Is it true?



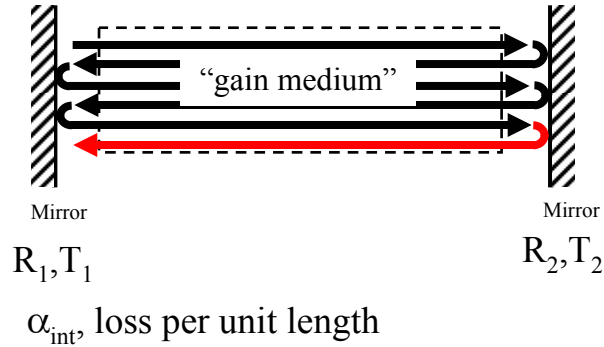
From Siegman, Lasers

# Effect of internal losses:



From Siegman, Lasers

# Rigrod analysis:



$$\frac{\partial I_+}{\partial z} = \gamma(z)I_+$$

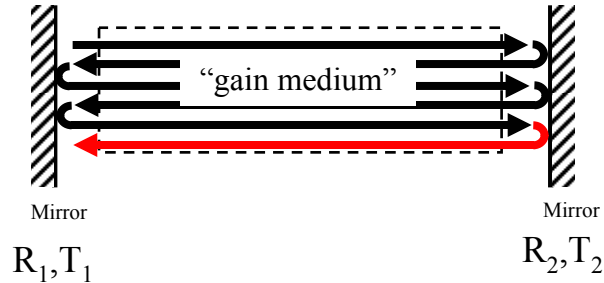
$$\frac{\partial I_-}{\partial z} = -\gamma(z)I_-$$

$$\frac{\partial}{\partial z} (I_+ \cdot I_-) = \frac{\partial I_+}{\partial z} \cdot (I_-) + \frac{\partial I_-}{\partial z} \cdot (I_+) = \gamma(z)I_+ \cdot (I_-) + -\gamma(z)I_- \cdot (I_+) = 0$$

$$\Rightarrow (I_+ \cdot I_-) = \text{a constant!} \equiv C$$

Reference: W.W. Rigrod, "Saturation effects in high-gain laser", *Journal of Applied Physics* **36**, 2487-2490 (August, 1965).

# Standing wave laser:



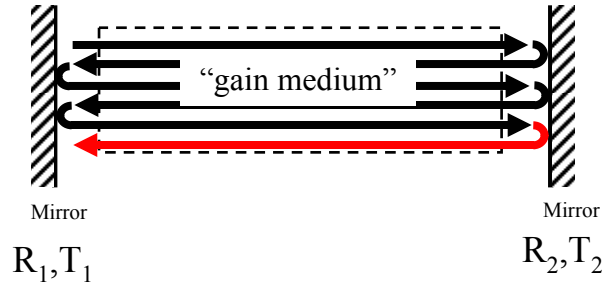
$$\gamma(z) = \frac{\gamma_0}{1 + (I_+(z) + I_-(z))/I_s} = \frac{\gamma_0}{1 + (I_+(z) + C/I_+(z))/I_s}$$

$$\frac{\partial I_+}{\partial z} = \gamma(z) I_+ \Rightarrow$$

$$\frac{\partial I_+}{\partial z} = \frac{\gamma_0}{1 + (I_+(z) + C/I_+(z))/I_s} I_+$$

Lets integrate and solve for  $I_+$

# Standing wave laser:



$$\frac{\partial I_+}{\partial z} = \frac{\gamma_0}{1 + \frac{C}{I_+(z) I_s}} I_+$$

$$\partial I_+ \left( \frac{1}{I_s} + \frac{1}{I_+} + \frac{C}{I_+^2 I_s} \right) = \gamma_0 \partial z$$

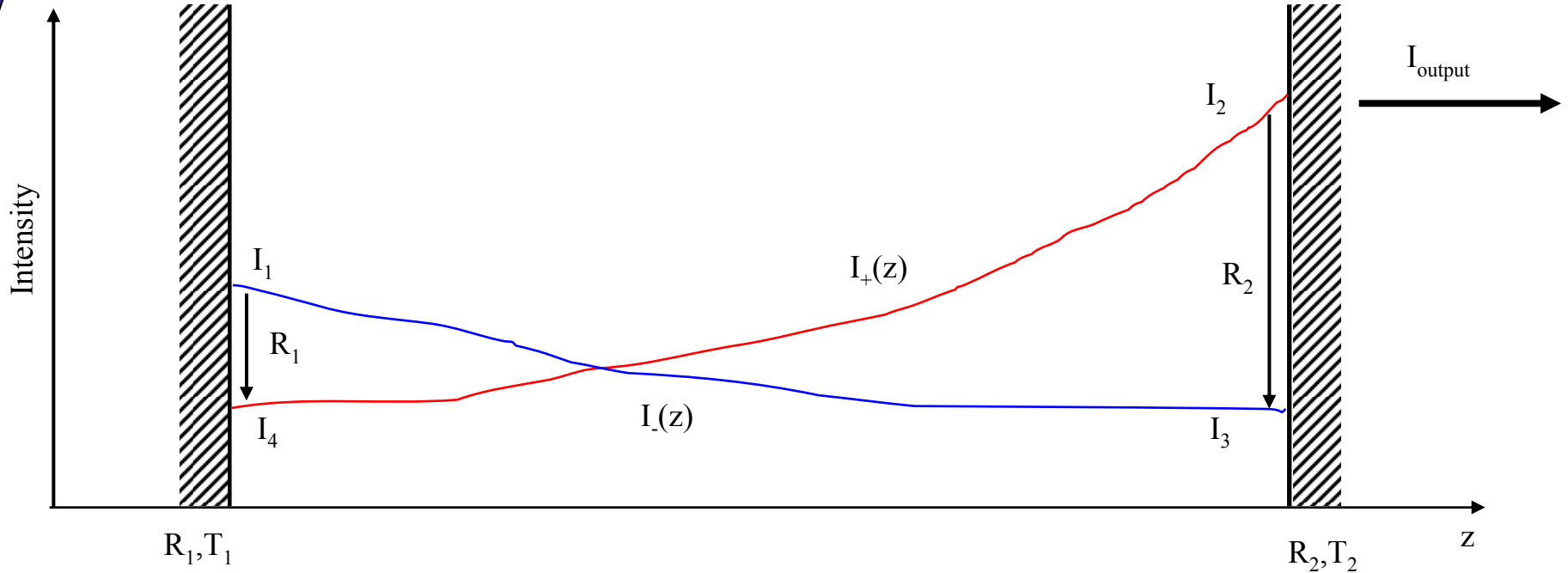
$$\int_{I_1}^{I_2} \partial I_+ \left( \frac{1}{I_s} + \frac{1}{I_+} + \frac{C}{I_+^2 I_s} \right) = \int_0^{l_g} \gamma_0 \partial z$$

$$\frac{I_2 - I_1}{I_s} + \ln \left( \frac{I_2}{I_1} \right) - \frac{C}{I_s} \left( \frac{1}{I_2} - \frac{1}{I_1} \right) = l_g \gamma_0$$

Do same thing for  $I_-$  in terms of  $I_3, I_4$ .

What are limits of integration?

# Integration limits:

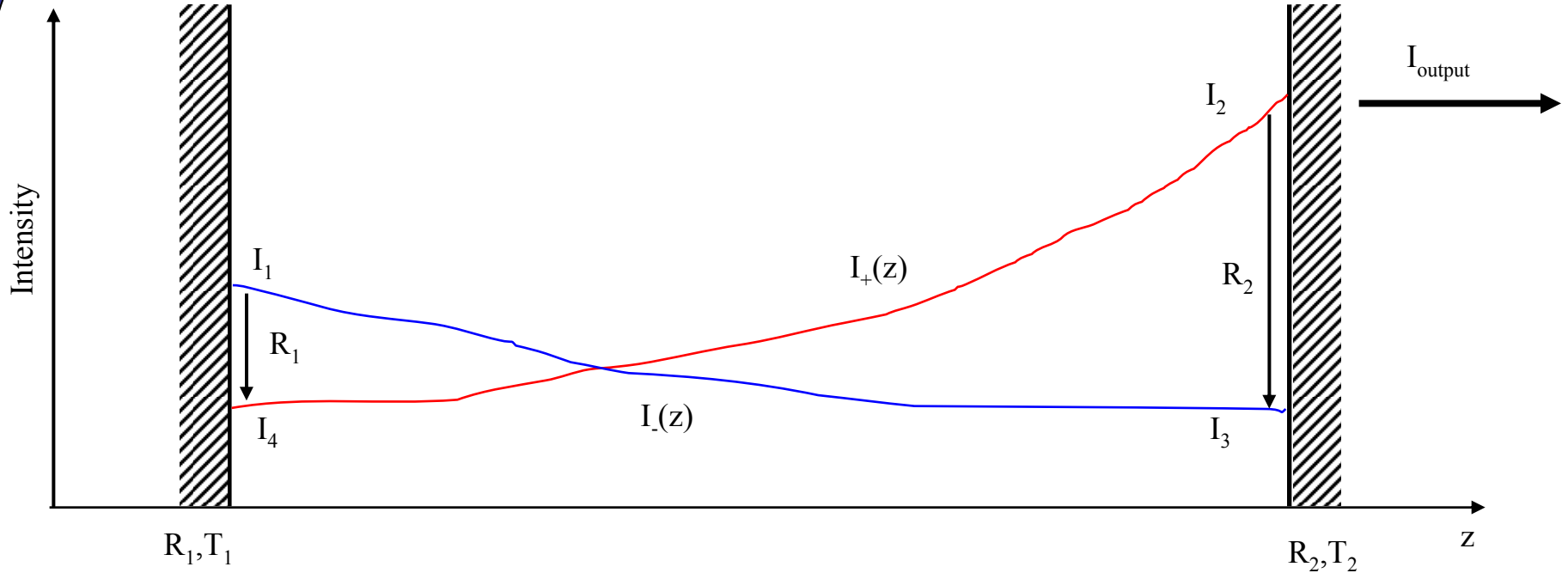


If you want the details, see Rigrod's paper.

Using these boundary conditions you can solve for the constant  $C$  and hence

$I_+$ ,  $I_-$ , and  $I_{\text{output}}$

# Integration limits:



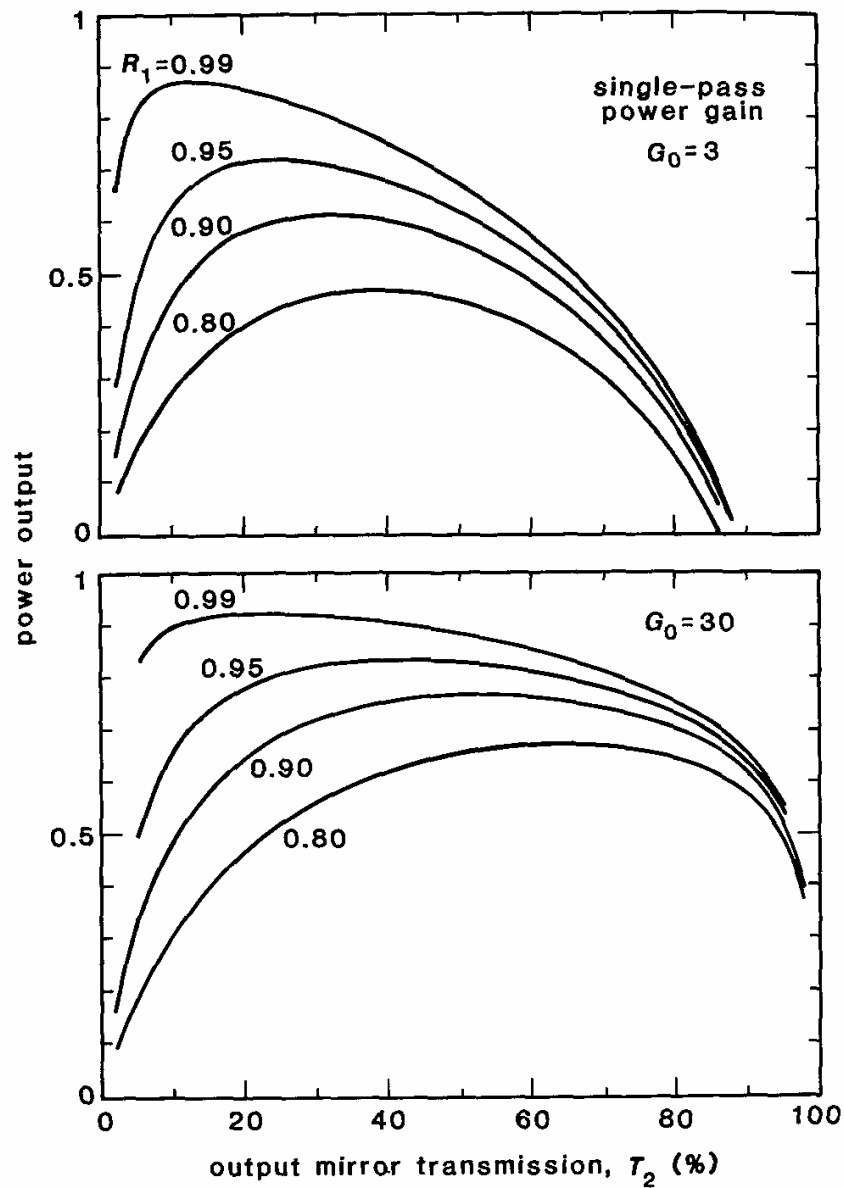
If you want the details, see Rigrod's paper.

Using these boundary conditions you can solve for the constant  $C$  and hence  $I_+$ ,  $I_-$ , and  $I_{\text{output}}$

$$I_{\text{output}} = T_2 \cdot I_2 = T_2 \cdot I_s \left[ \frac{\gamma_0 l_g - 0.5 \cdot \ln(R_1 R_2)}{\left(1 - \sqrt{R_1 R_2}\right) \left(1 + \sqrt{R_2 / R_1}\right)} \right]$$

This covers all cases. Must be evaluated numerically for typical examples.

# Typical example:



From Siegman, Lasers

That concludes our  
discussion of laser  
oscillation in the steady  
state.

# On the horizon

- Time-dependent laser oscillation (Q-switching, mode-locking)
- Example laser systems
- Beams and propagation in fibers
- Semiconductor lasers