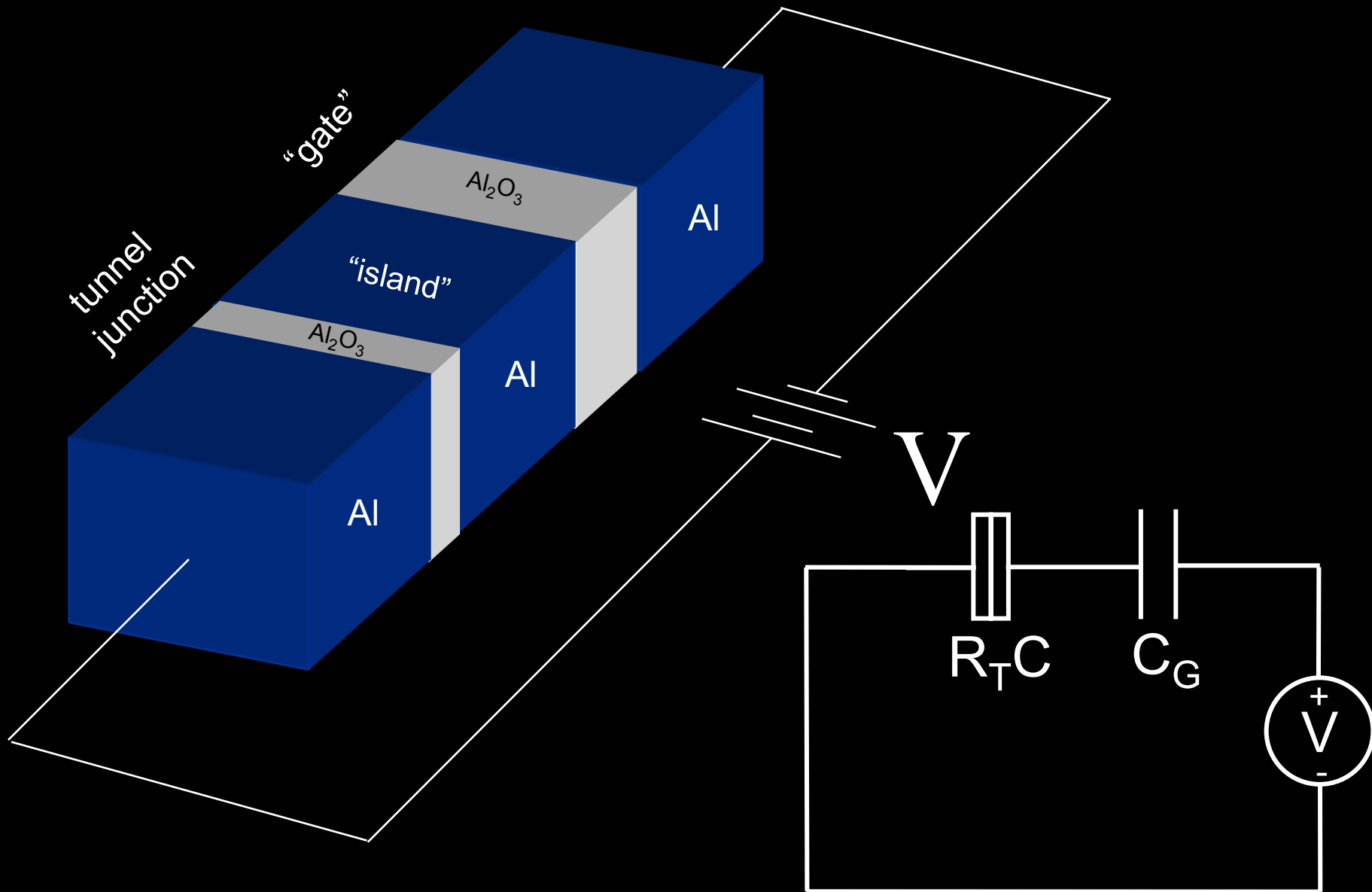
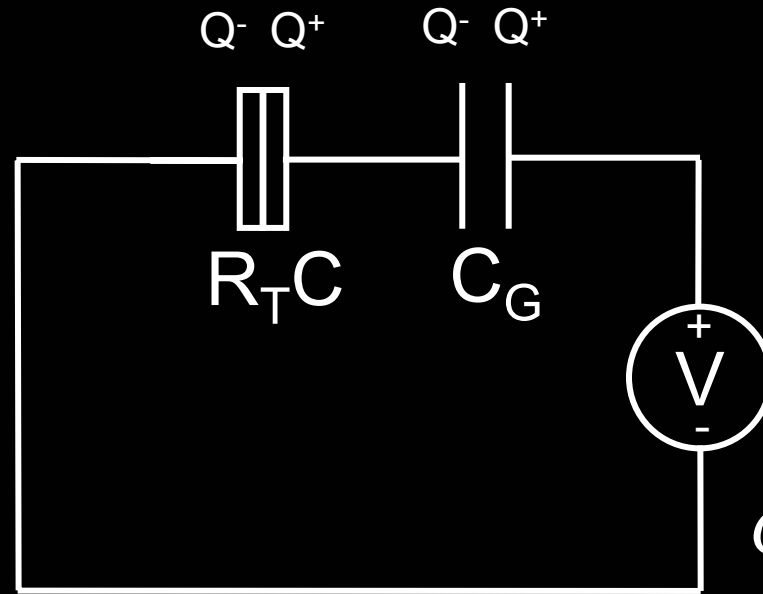


Lecture 6: Single electron box



Electrostatic energy (*no tunneling*)

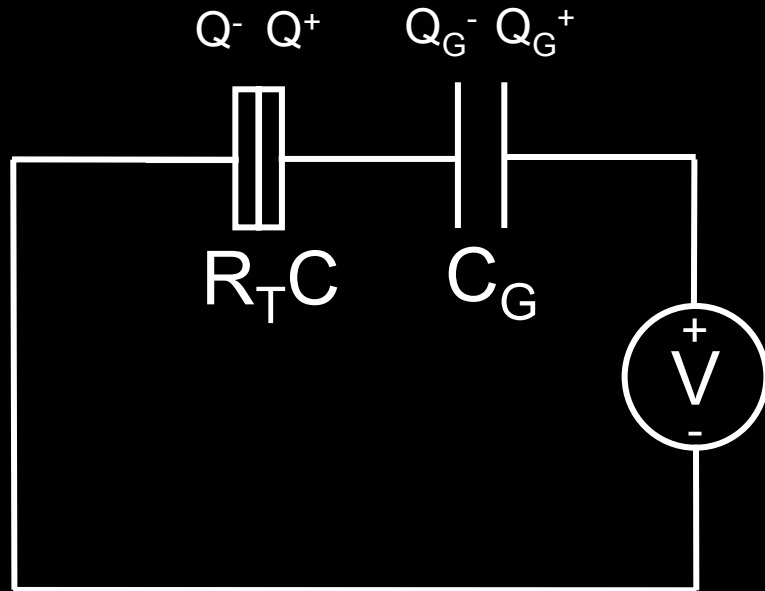


$$C_{series}^{-1} \equiv C^{-1} + C_G^{-1}$$

$$V = \frac{Q}{C} + \frac{Q}{C_G} = Q \left(\frac{1}{C} + \frac{1}{C_G} \right) = \frac{Q}{C_{series}}$$

$$E = \frac{Q^2}{2C} + \frac{Q^2}{2C_G} = \frac{Q^2}{2C_{series}} = \frac{1}{2} C_{series} V^2$$

Island charge



“Island charge”:

$$Q_i = Q - Q_G$$

Kirchoff:

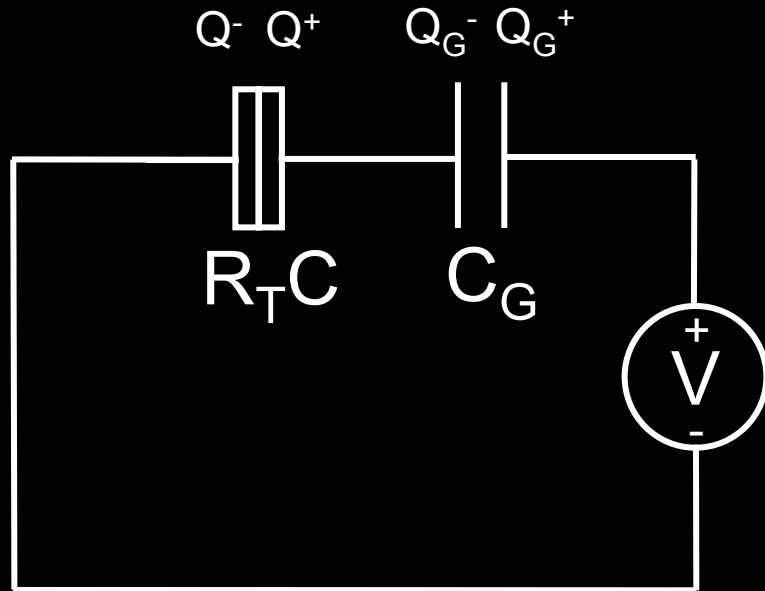
$$V = \frac{Q}{C} + \frac{Q_G}{C_G}$$

Solve for Q , Q_G :

$$Q = \frac{C(C_G V + Q_i)}{C + C_G}$$

$$Q_G = \frac{C_G(CV - Q_i)}{C + C_G}$$

Electrostatic energy (*with tunneling*)



$$Q = \frac{C(C_G V + Q_i)}{C + C_G}$$

$$Q_G = \frac{C_G(CV - Q_i)}{C + C_G}$$

$$E = \frac{Q^2}{2C} + \frac{Q_G^2}{2C_G} = \frac{CC_G V^2 + Q_i^2}{2(C + C_G)}$$

Thermodynamics

Entropy:

$$S = S(E, V, N, \dots)$$

Energy:

$$E = E(S, V, N, \dots)$$

Entropy maximum \Leftrightarrow Energy minimum

Thermodynamic variables

Energy:

$$E = E(S, V, N, \dots)$$

Temperature:

$$\frac{1}{T} \equiv \left. \frac{\partial E}{\partial S} \right|_{V, N, \dots}$$

Pressure:

$$P \equiv - \left. \frac{\partial E}{\partial V} \right|_{E, N, \dots}$$

Thermodynamic potentials

Energy:

$$E = E(S, V, N, \dots)$$

Helmholtz potential (Helmholtz free energy):

$$F = E - TS$$

Minimized in presence of
“reservoir” with temperature T .

Enthalpy:

$$H = E + PV$$

Minimized in presence of
“reservoir” with pressure P .

Gibbs free energy:

$$G = E - TS + PV$$

Minimized in presence of
“reservoir” with pressure P ,
temperature T .

Thermodynamic potentials for circuits


Energy:

$$E = E(S, V, N, Q, \dots)$$

Gibbs free energy for electronic circuits:

$$G = E - Q_G V$$

Minimized in presence of
"reservoir" with voltage V .



electrostatic
energy



need to calculate

Q = how much charge has passed through the battery onto the gate

V = voltage of the battery

Free energy of single electron box:

$$G = E - Q_G V$$

From before:

$$E = \frac{CC_G V^2 + Q_i^2}{2(C + C_G)}$$

$$Q_G = \frac{C_G(CV - Q_i)}{C + C_G}$$

$$G = \frac{CC_G V^2 + Q_i^2}{2(C + C_G)} - \frac{C_G(CV - Q_i)}{C + C_G} V$$

$$= \frac{1}{2} \frac{(C_G V + Q_i)^2}{C + C_G} - \frac{1}{2} C_G V^2$$

(Note: The last minus sign agrees with Lafarge thesis, but not Ferry textbook.)

Charge of island

From last slide:

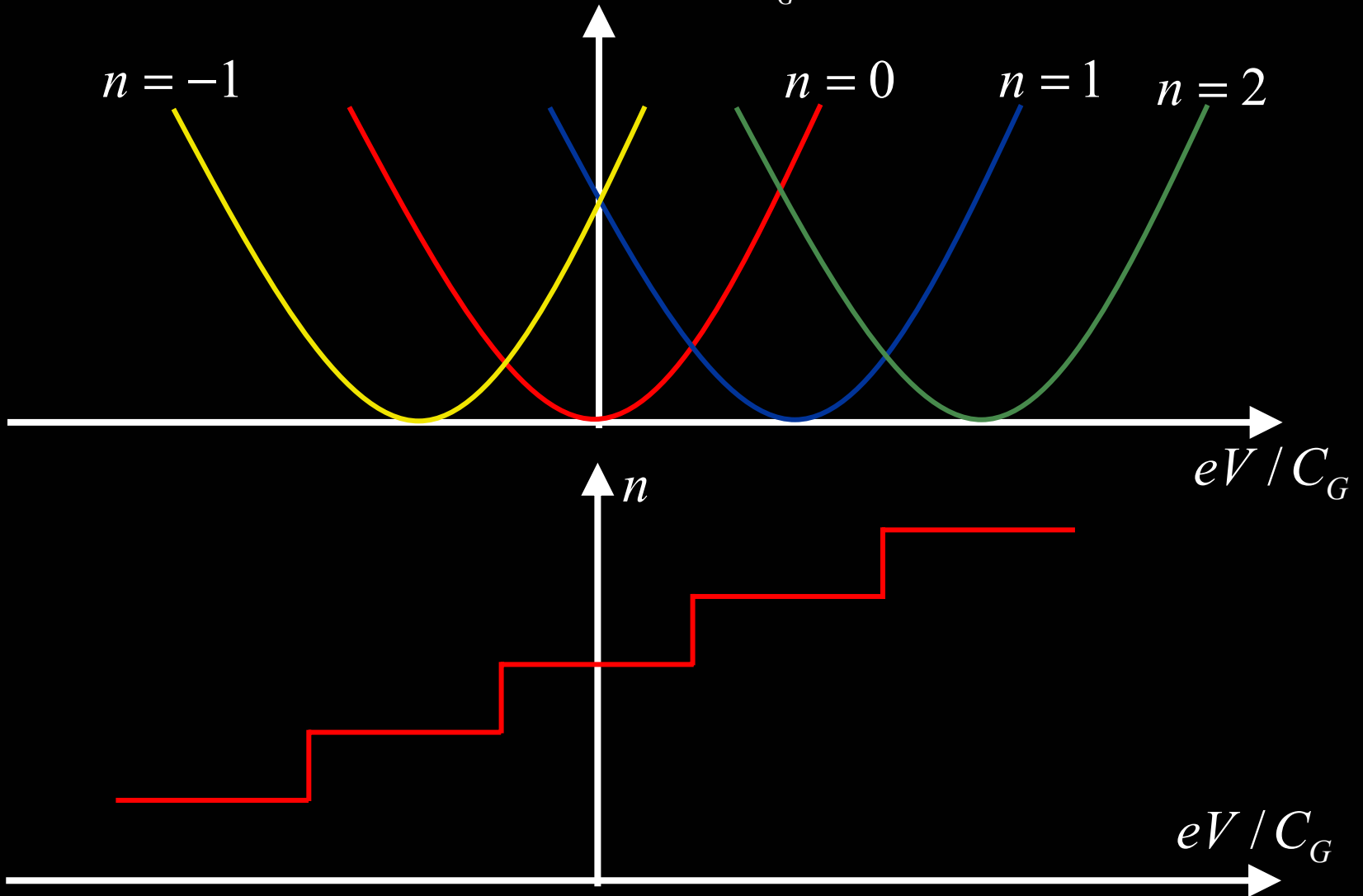
$$G = \frac{1}{2} \frac{(C_G V + Q_i)^2}{C + C_G} - \frac{1}{2} C_G V^2$$

$$Q_i = -ne \quad \text{only if } R_T \gg R_K$$

$$G = \frac{1}{2} \frac{(C_G V - ne)^2}{C + C_G} + \text{const}$$

Charge of island

$$G = \frac{1}{2} \frac{(C_G V - ne)^2}{C + C_G}$$



Finite temperatures

Need
 $kT \ll e^2/(C+C_G)$

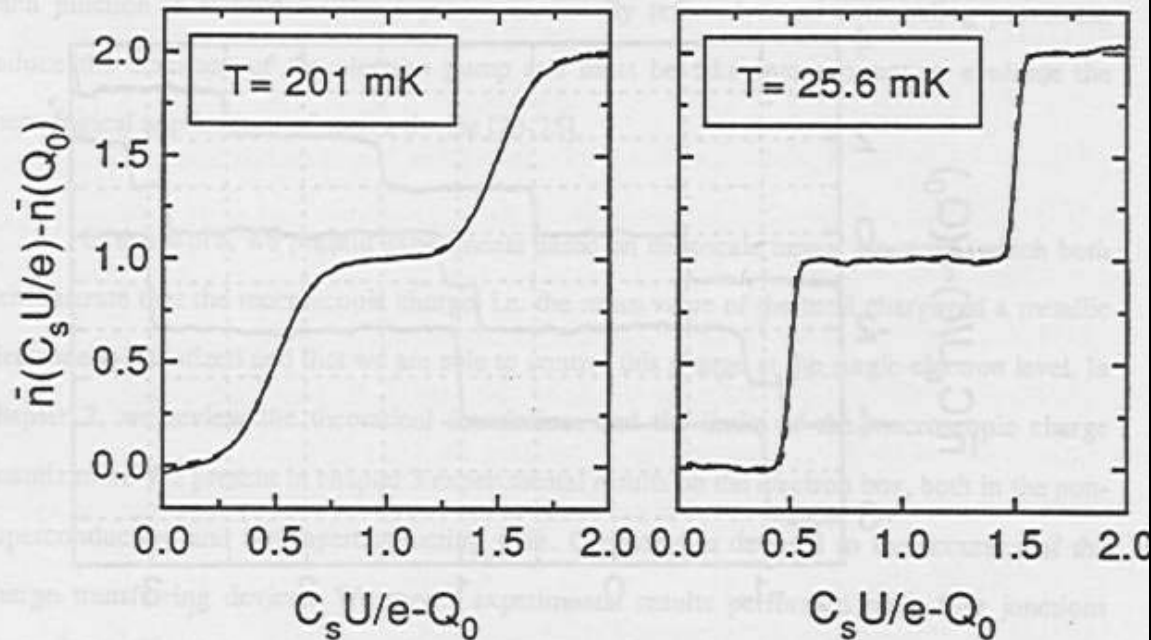


Fig. 1.5 b) Solid lines: experimental variations of the average number \bar{n} of excess electrons in the island of an electron box. Dashed lines: theoretical calculations for an island capacitance $C_\Sigma = 0.8 \text{ fF}$. The experimental parameters of the circuit are $C_s = 74 \text{ aF}$ and $C_c = 21 \text{ aF}$. The quantity Q_0 denotes the random offset charge in the island.

From Lafarge, PhD thesis, Universite Paris 6 (1993)

Quantum computing

- A single electron box has been proposed as a qu-bit
- $|0\rangle$ or $|1\rangle$ correspond to n or $n+1$ electrons
- Difficulty is fast (GHz) readout before decoherence sets in
- A superconducting box (for Cooper pairs) could have longer decoherence