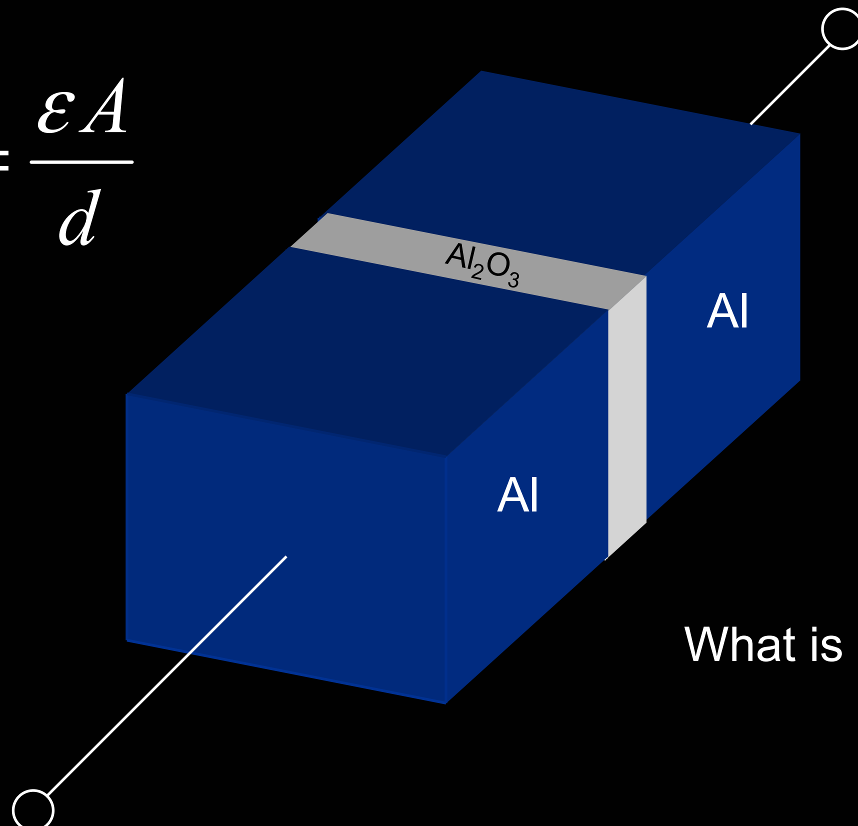
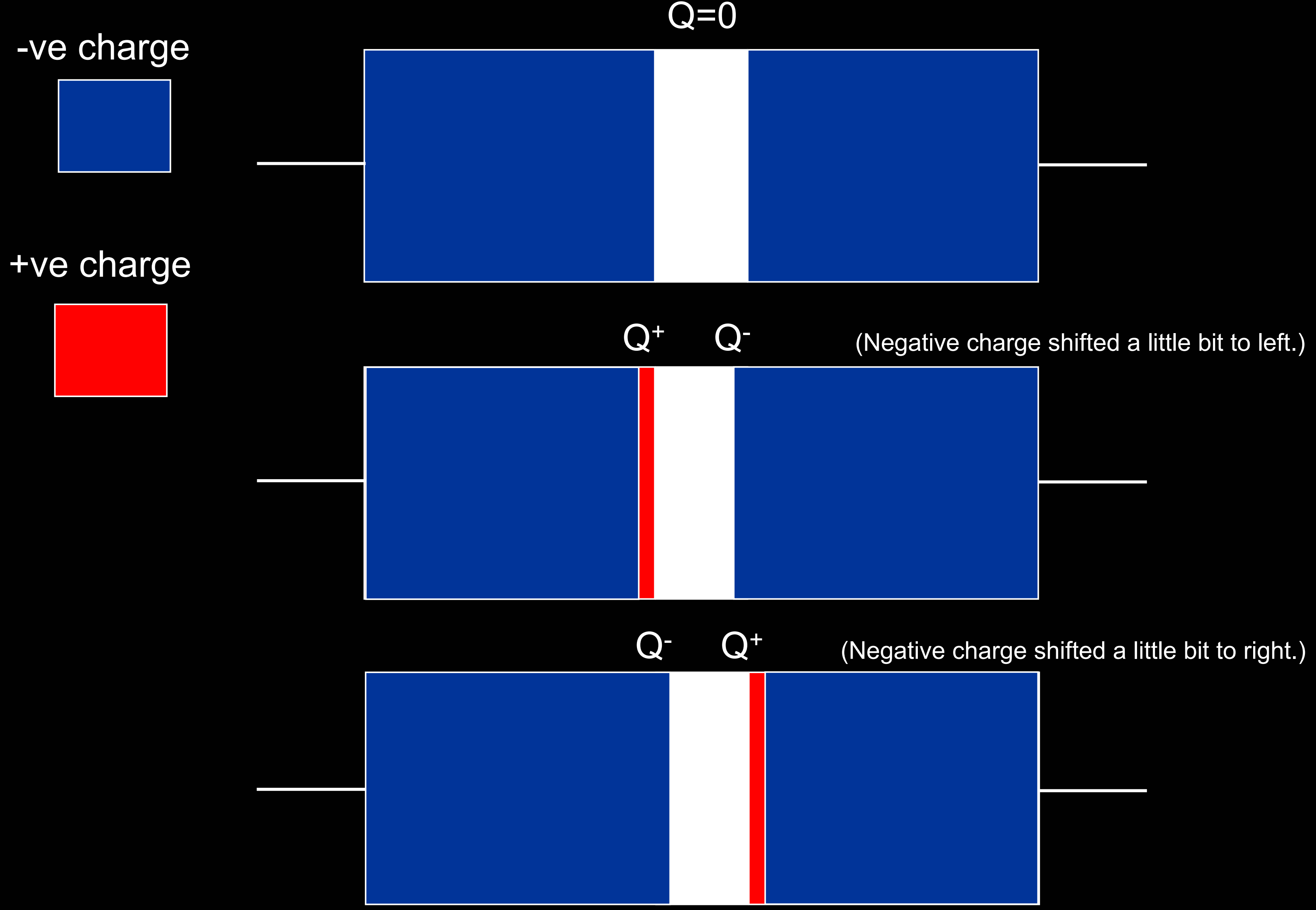
# Lecture 5: Coulomb blockade

$$C = \frac{\varepsilon A}{d}$$




What is charge on this capacitor?

# Charge on capacitor continuous

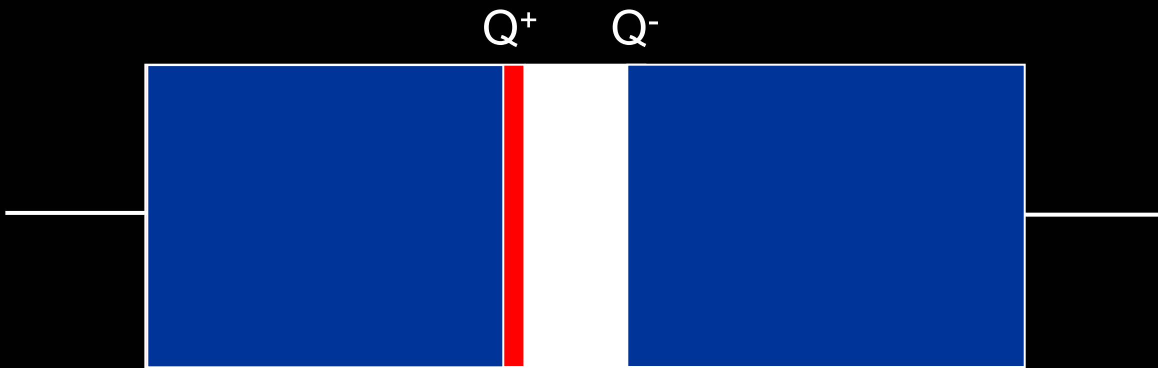


-ve charge

+ve charge

# Is tunneling allowed?

(Negative charge shifted a little bit to left.)




$$E = \frac{Q^2}{2C}$$

-ve charge

+ve charge

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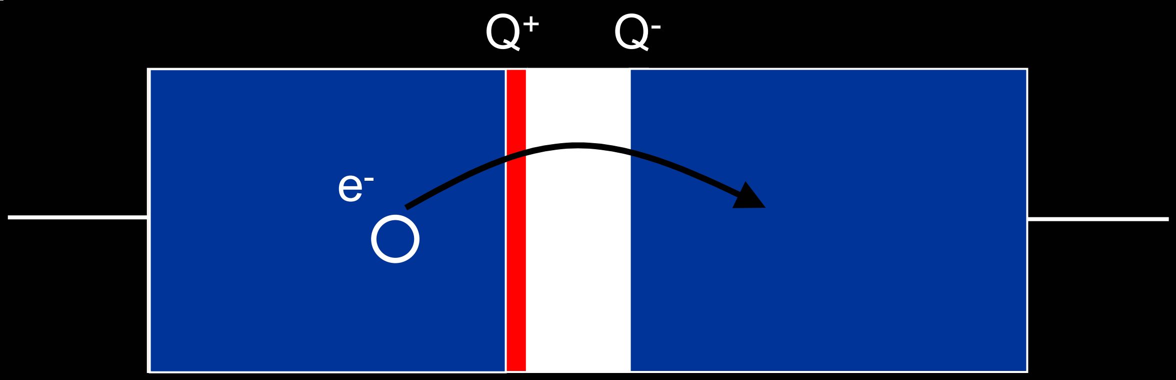


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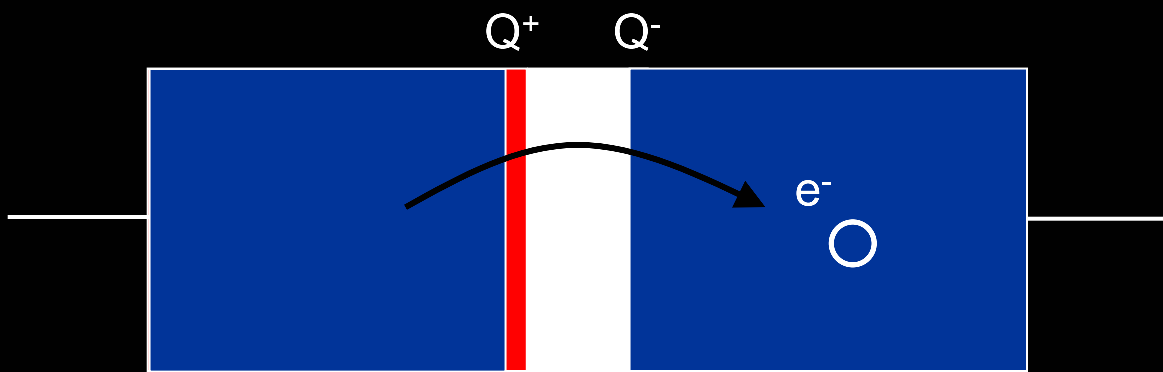
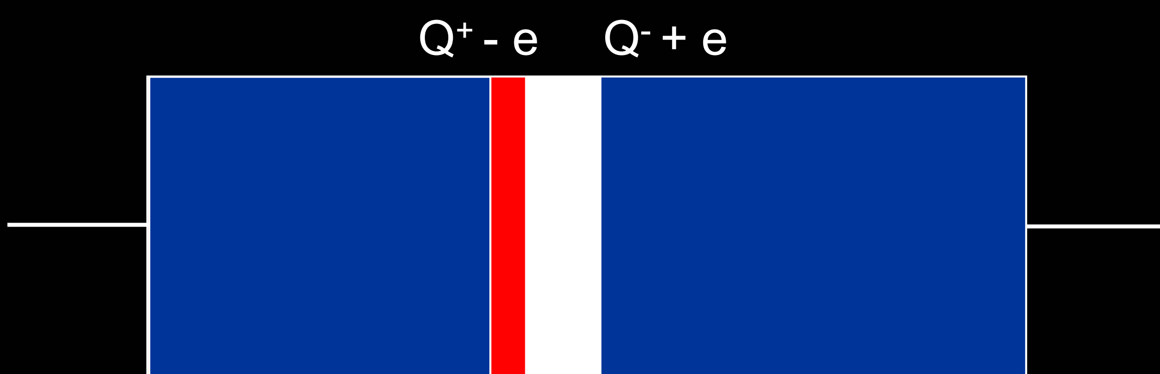
(Negative charge shifted a little bit to left.)




$$E = \frac{Q^2}{2C}$$

After electron tunnels:




$$E = \frac{(Q-e)^2}{2C}$$

$$\Delta E = \frac{e(Q-e/2)}{C}$$

-ve charge

+ve charge

# Coulomb gap

$$\Delta E = \frac{e(Q - e/2)}{C} > 0$$

$$\Rightarrow Q - e/2 > 0$$

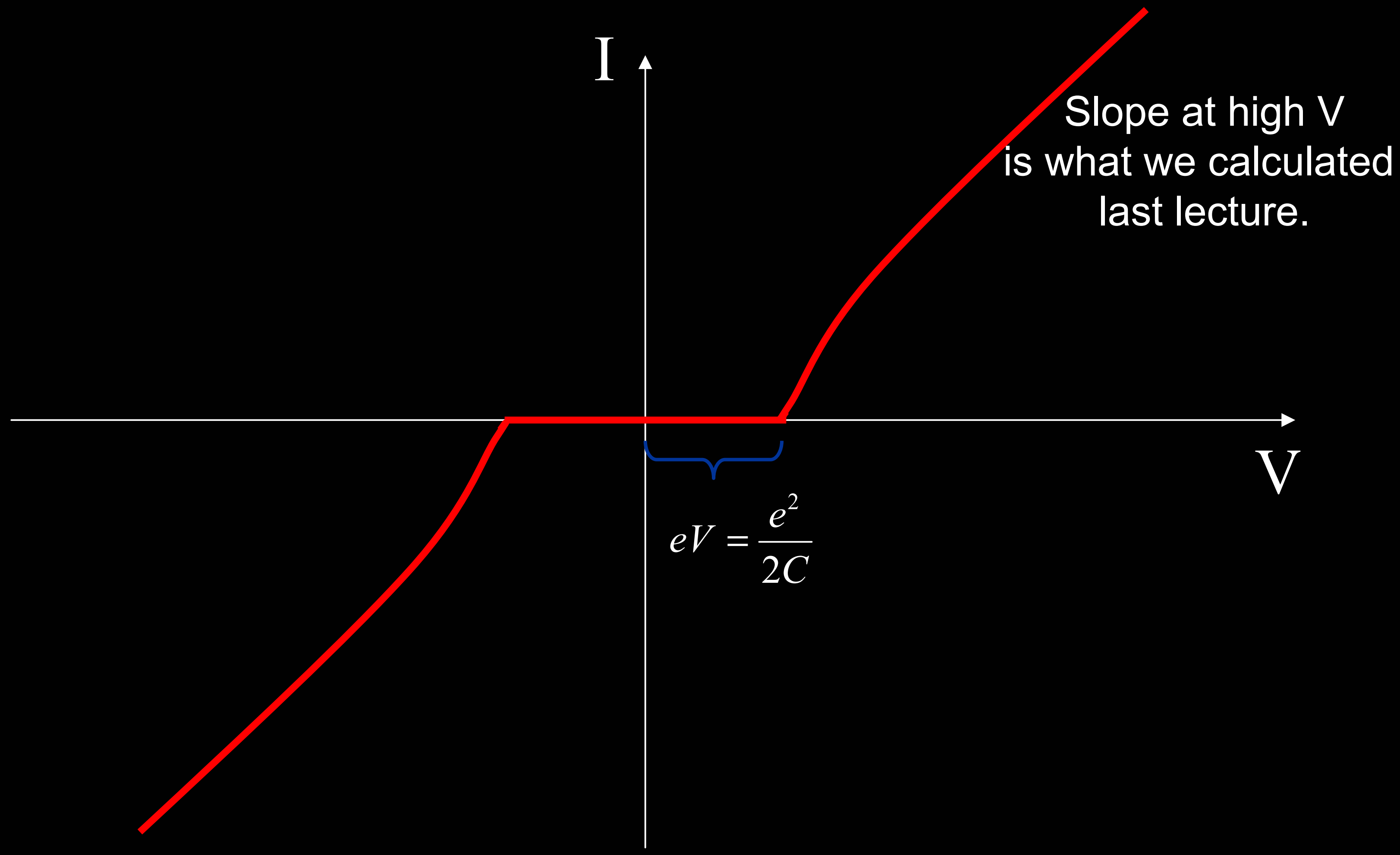
$$\Rightarrow Q > e/2$$

$$\Rightarrow C|V| > e/2 \Rightarrow |V| > \frac{e}{2C}$$

$$\boxed{-\frac{e}{2C} < V < \frac{e}{2C}}$$

Tunneling only under these conditions, *otherwise no tunneling!*

# I-V curve



I
V
$eV = \frac{e^2}{2C}$
Slope at high V is what we calculated last lecture.

# Temperature

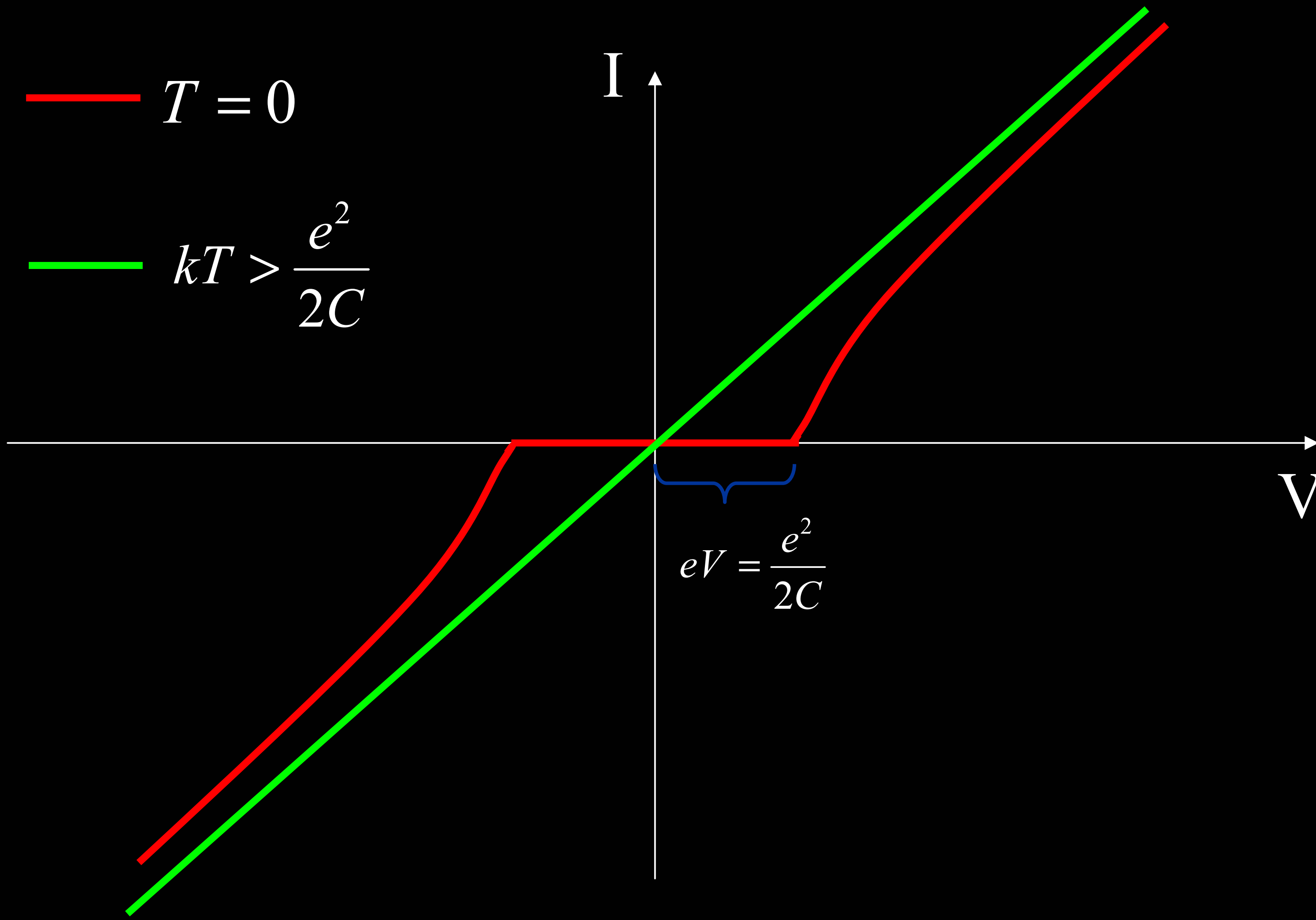
$$\Delta E = \frac{e(Q - e/2)}{C}$$

can be less than 0 if thermal energy available

Criteria to observe coulomb gap behavior:

$$\frac{e^2}{C} > kT$$

# I-V curve vs. temperature



# Numbers

Last class demo:
1 nm barrier, 1 mm x 1 mm junction:

$$C = \frac{\varepsilon A}{d} = \frac{10 \cdot 8.85 \cdot 10^{-12} F/m (10^{-3} m)^2}{10^{-9} m} \approx 10^{-7} F$$

$$\frac{e^2}{C} > kT \Rightarrow T < \frac{e^2}{Ck} = \frac{(1.6 \cdot 10^{-19} coulomb)^2}{10^{-7} F \cdot 1.38 \cdot 10^{-23} J/K} \approx 10^{-8} K$$

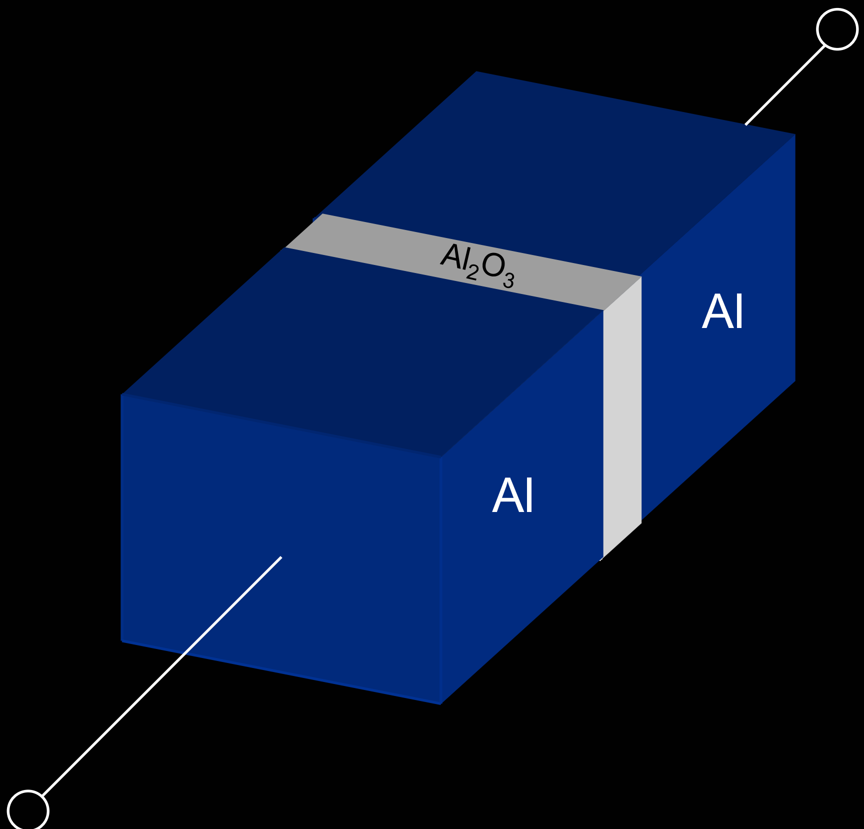
Practically impossible.

Best lithographic junction:
1 nm barrier, 100 nm x 100 nm junction:

$$C \approx 10^{-15} F \Rightarrow T < 1K$$

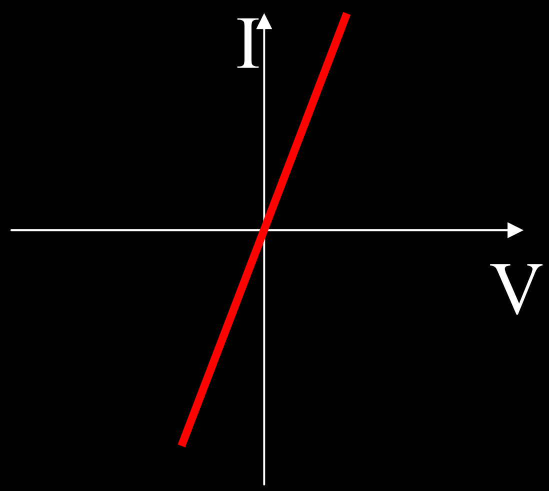
Possible to achieve in the lab.

# Quantum mechanics



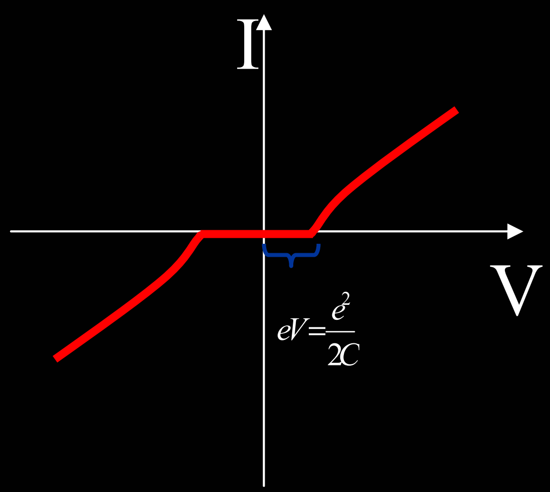

- For strong tunneling, electron can have a large probability to be on both sides at the same time.
- This means the system energy cannot be defined by localizing the electron on only one side.
- This makes coulomb blockade irrelevant.

# I-V curve vs. tunnel strength




Strong tunneling

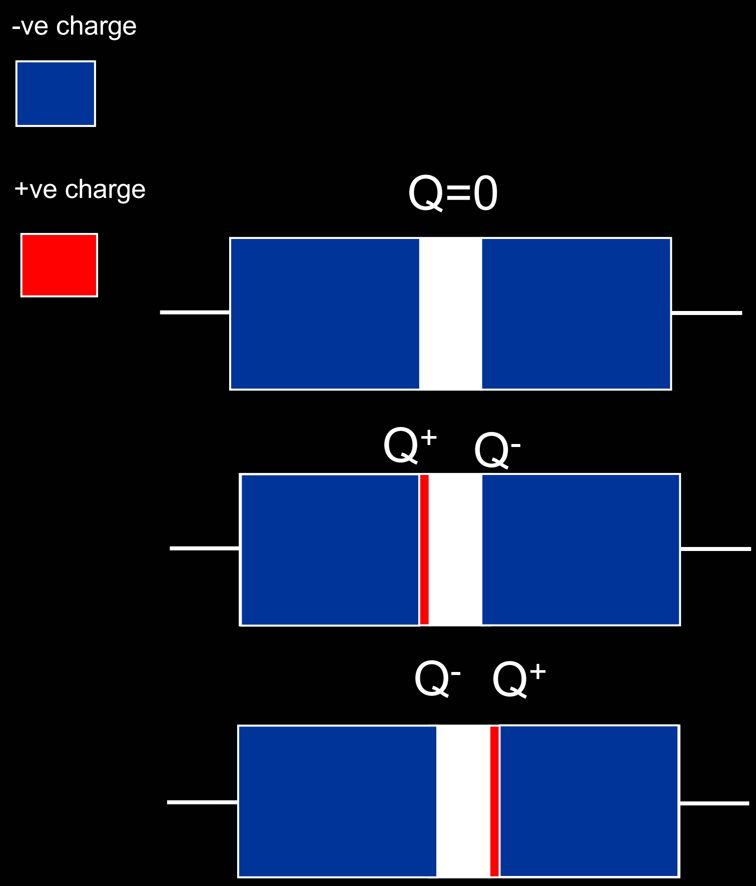
$$R_T < R_K \equiv \frac{h}{e^2} = 25\ \mathrm{k}\Omega$$




Weak tunneling

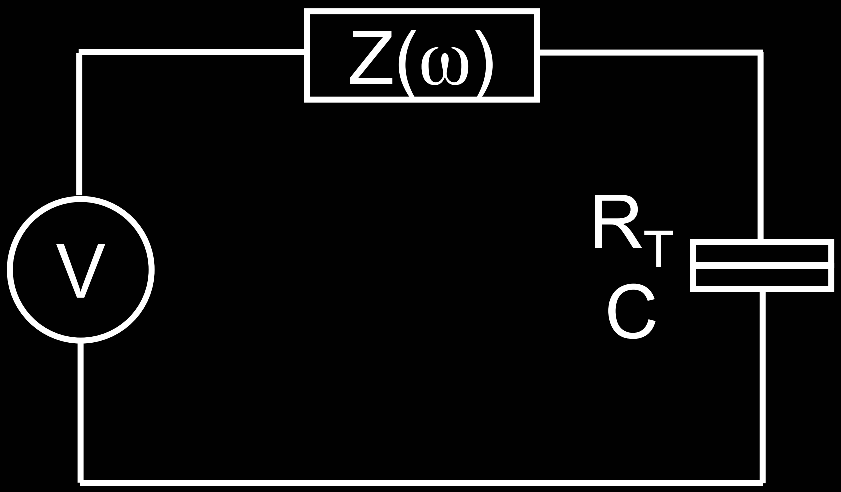
$$R_T > \frac{h}{e^2} = 25\ \mathrm{k}\Omega$$

# Charge on capacitor is a *quantum variable*




- We don't always know what Q is.
- Treating Q as a quantum variable, there is a certain probability for the system to have a certain value of Q.
- Should describe a "wave function" for Q: $\Psi(Q)$ just like wave function for position $\Psi(x)$
- Now, we need quantum theory of electric circuits.

# Quantum theory of electric circuits




- At DC, can have current bias or voltage bias depending on Z(dc) vs. $R_T$.
- At AC, almost always have $Z(\omega) < R_T$ because of lead capacitance (typically pF).

Full quantum treatment beyond the scope of this class.

In order to see Coulomb blockage,
need current bias all the way up to $1/(R_K C)$ which is typically 10 GHz, i.e.:

$$Z(\omega) > R_T \text{ for all } \omega \leq \frac{1}{R_K C} \sim 10 \text{ GHz}$$

# Requirements for Coulomb blockade

- $kT < e^2/C$ (hard)
- $R_T > R_K$ (25 kΩ) (harder)
- $Z(\omega) > R_T$ at all frequencies up to $1/R_KC$ (hardest)

Achieved by Cleland PhD thesis, Berkeley 1992.
(Congratulations, Andrew.)