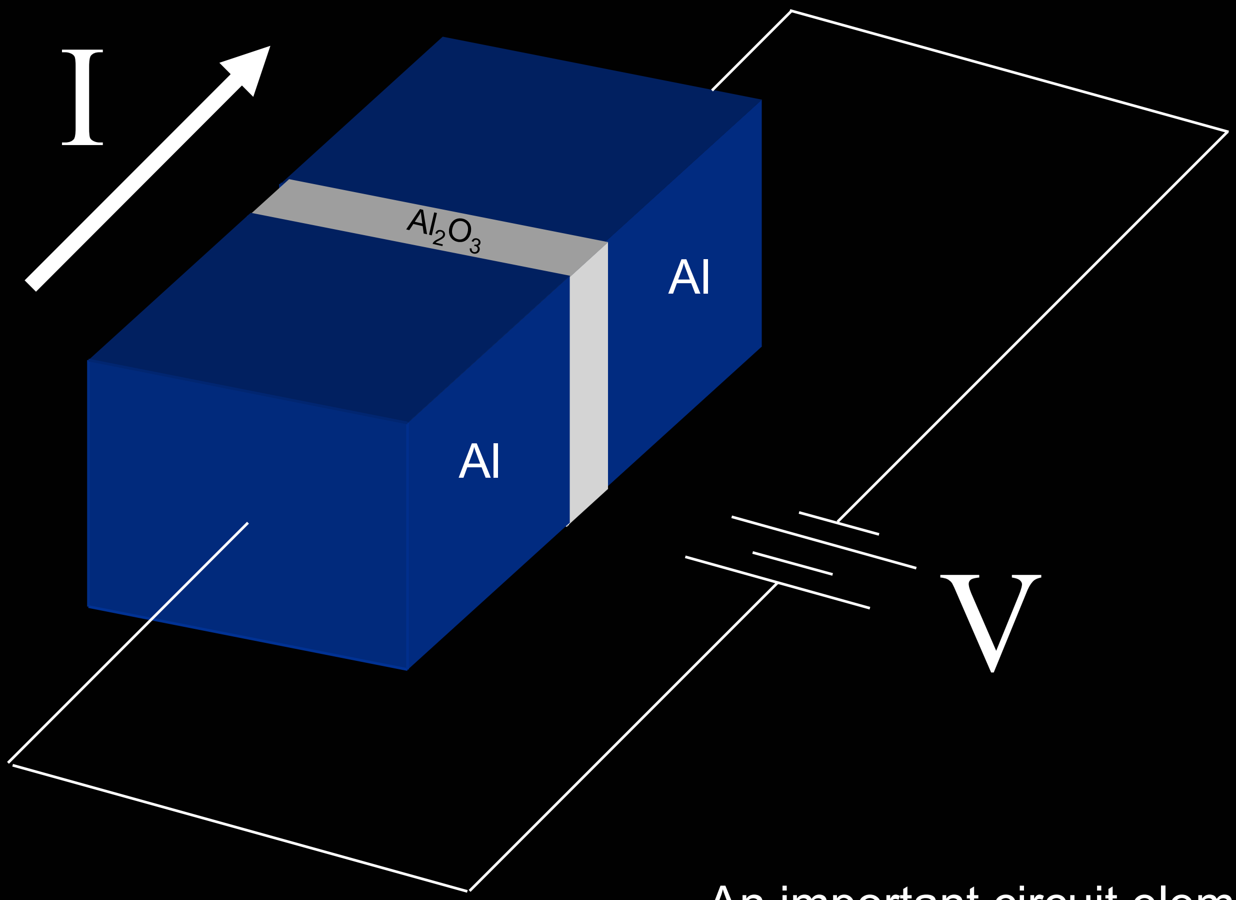
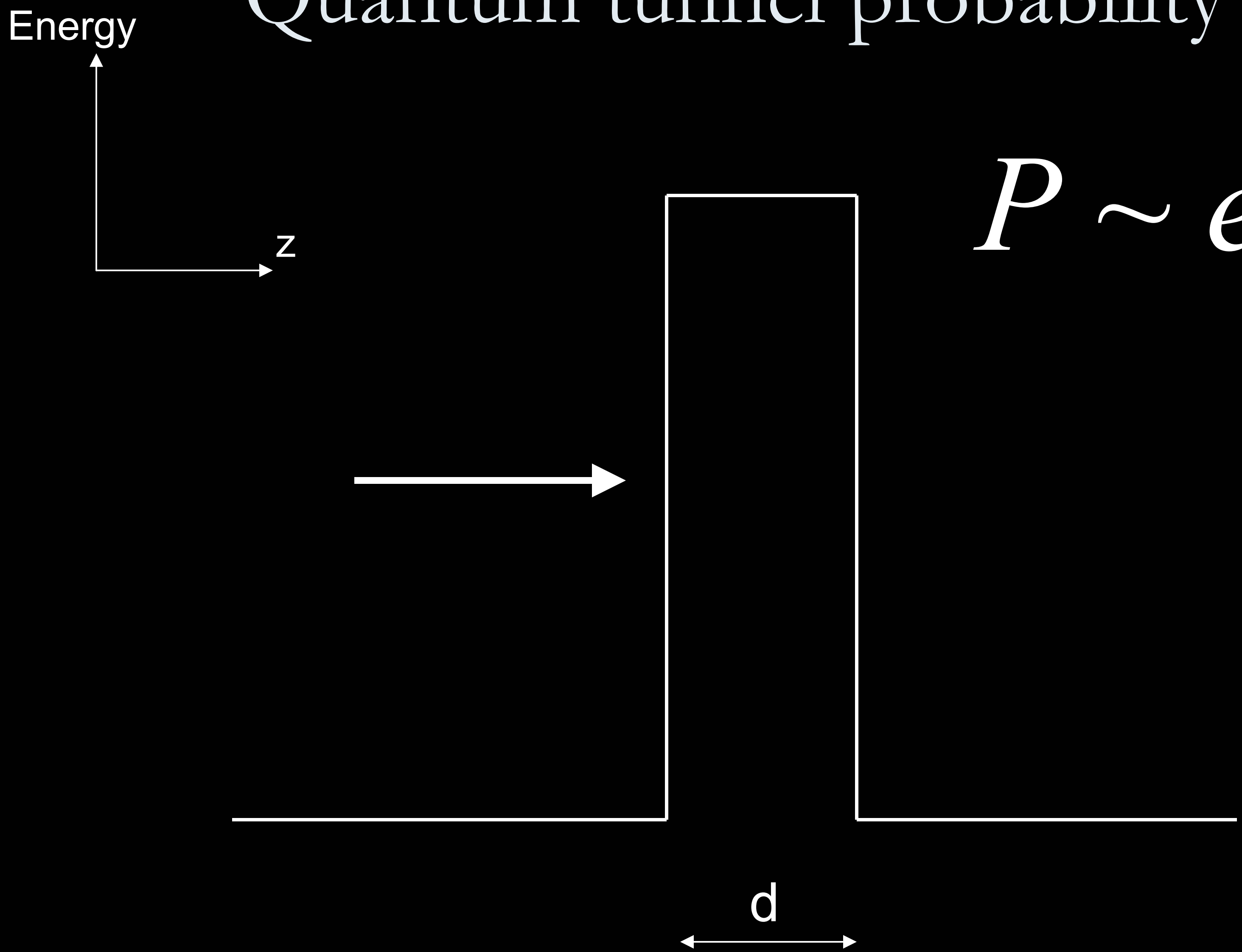
Tunnel junctions



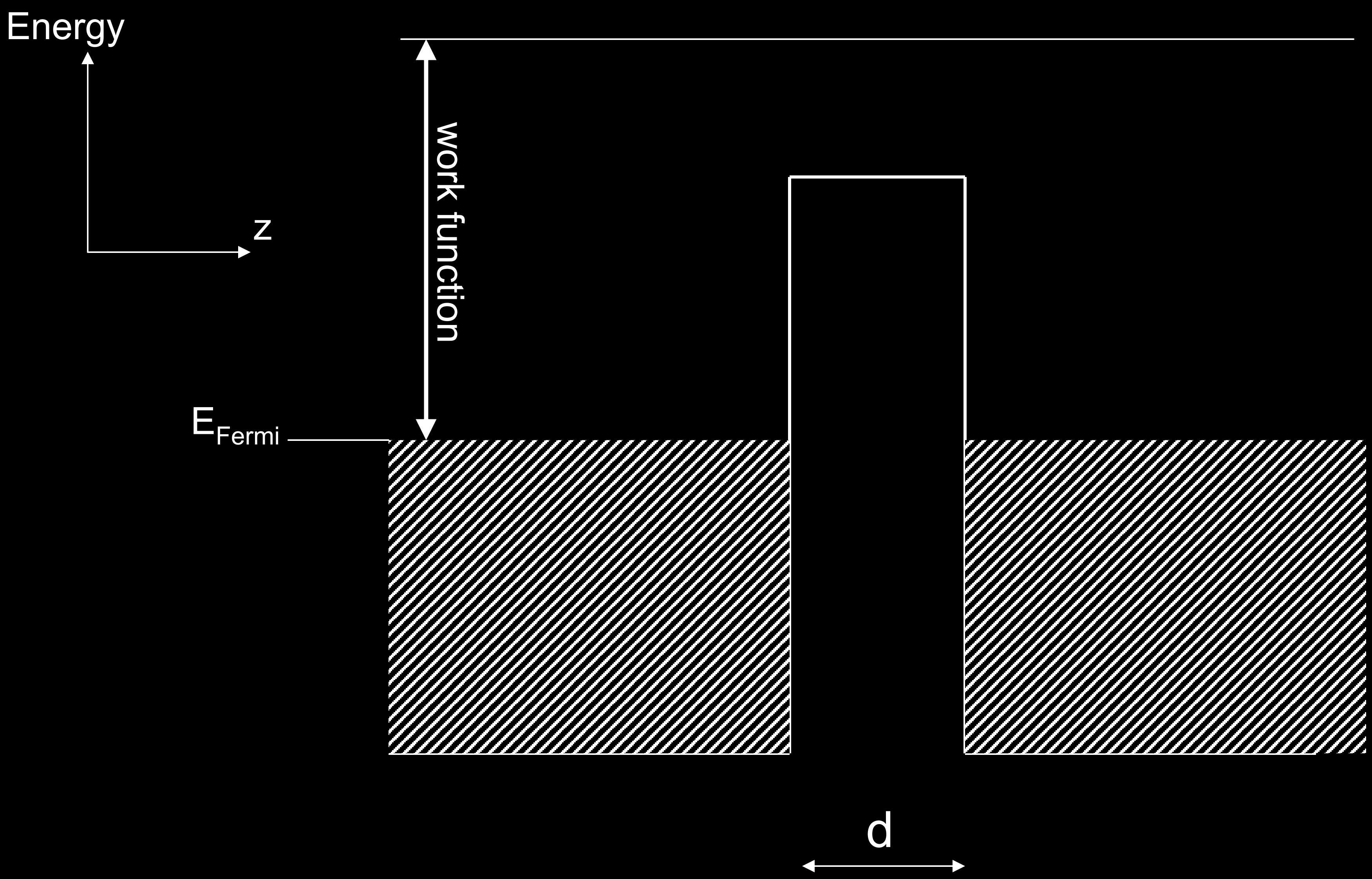
An important circuit element in *single electron transistors*.

Quantum tunnel probability

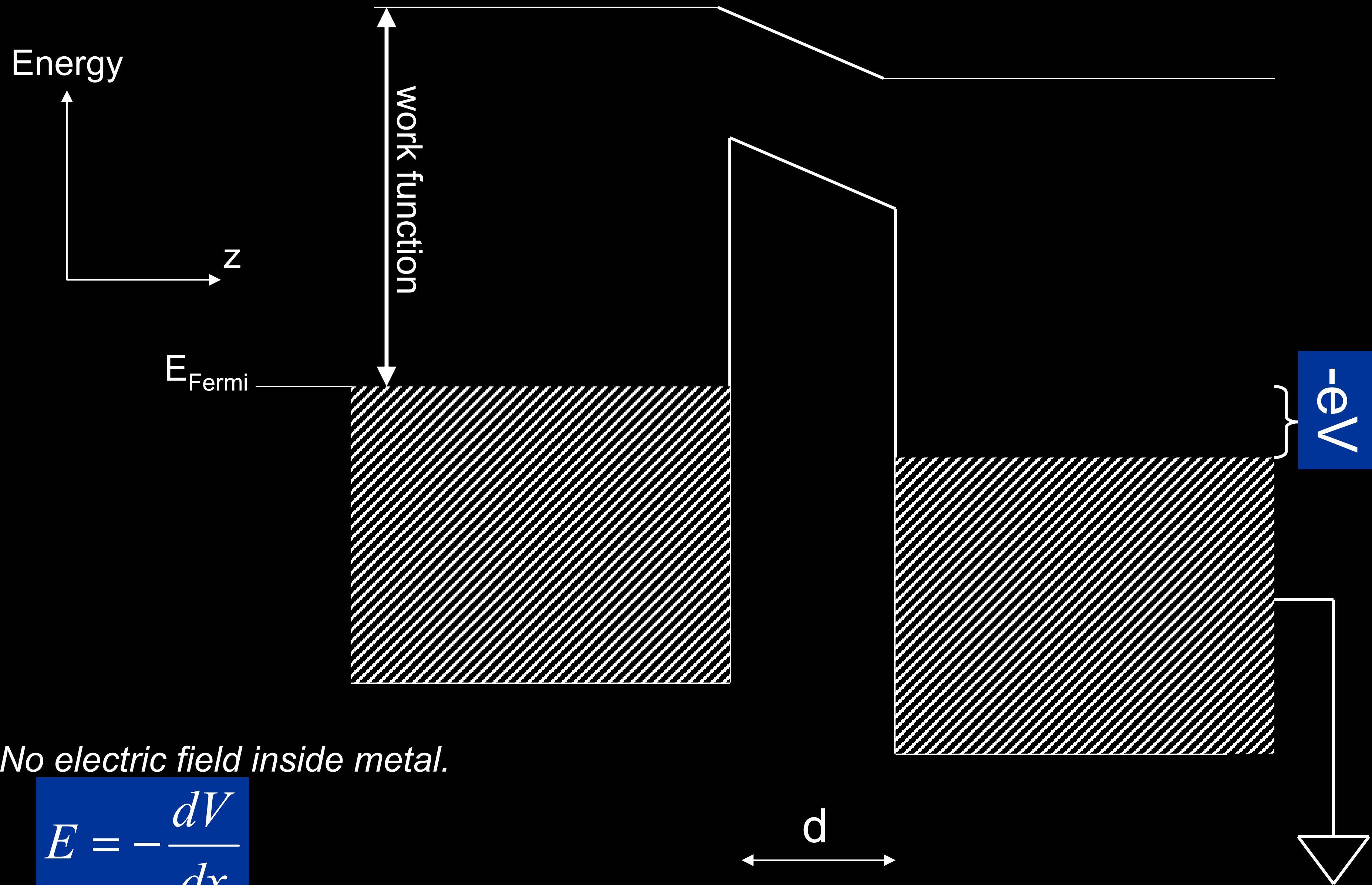


$$P \sim e^{-d}$$

Band diagram for tunnel junction



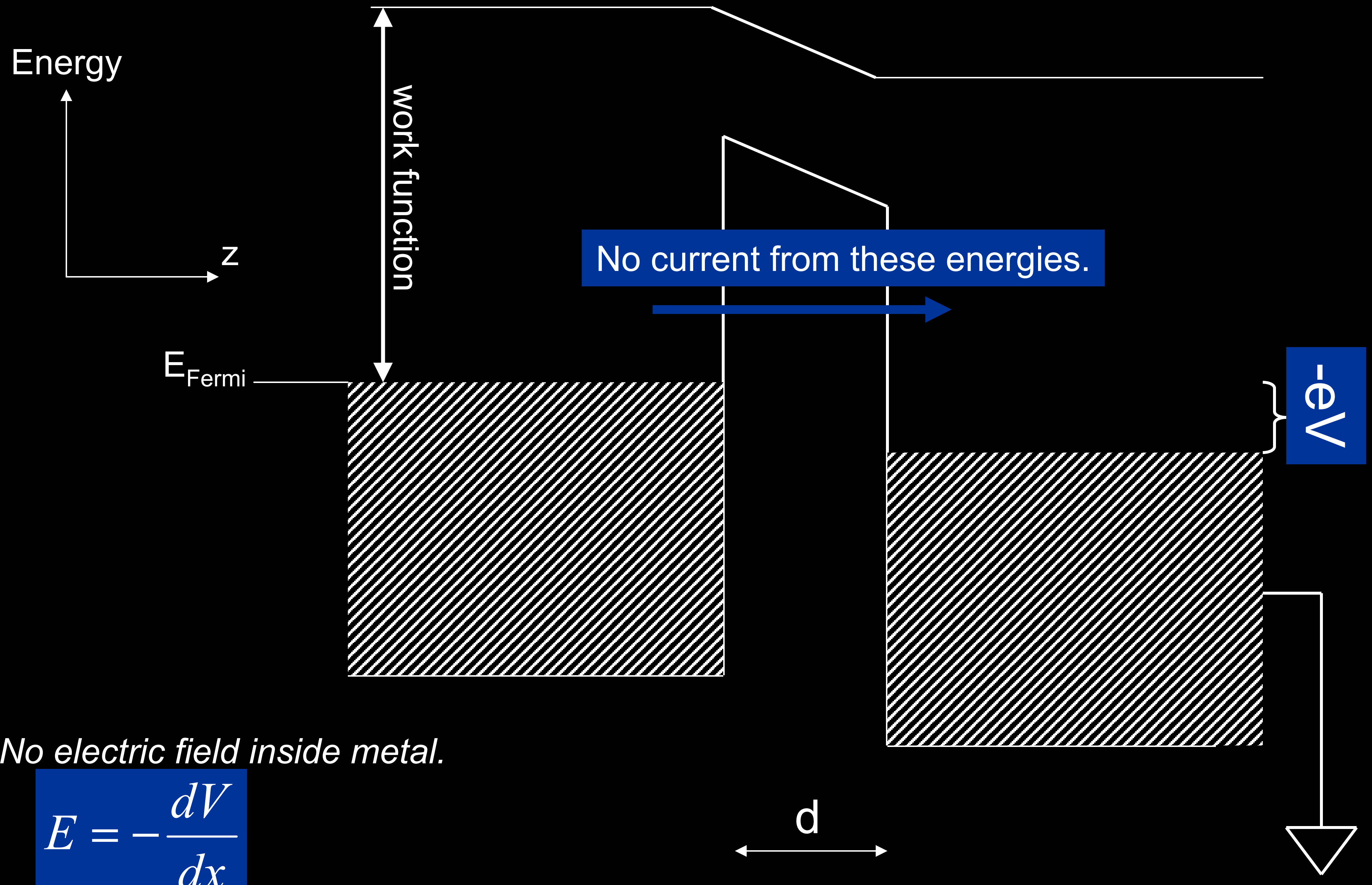
Band diagram under bias



No electric field inside metal.

$$E = -\frac{dV}{dx}$$

Band diagram under bias



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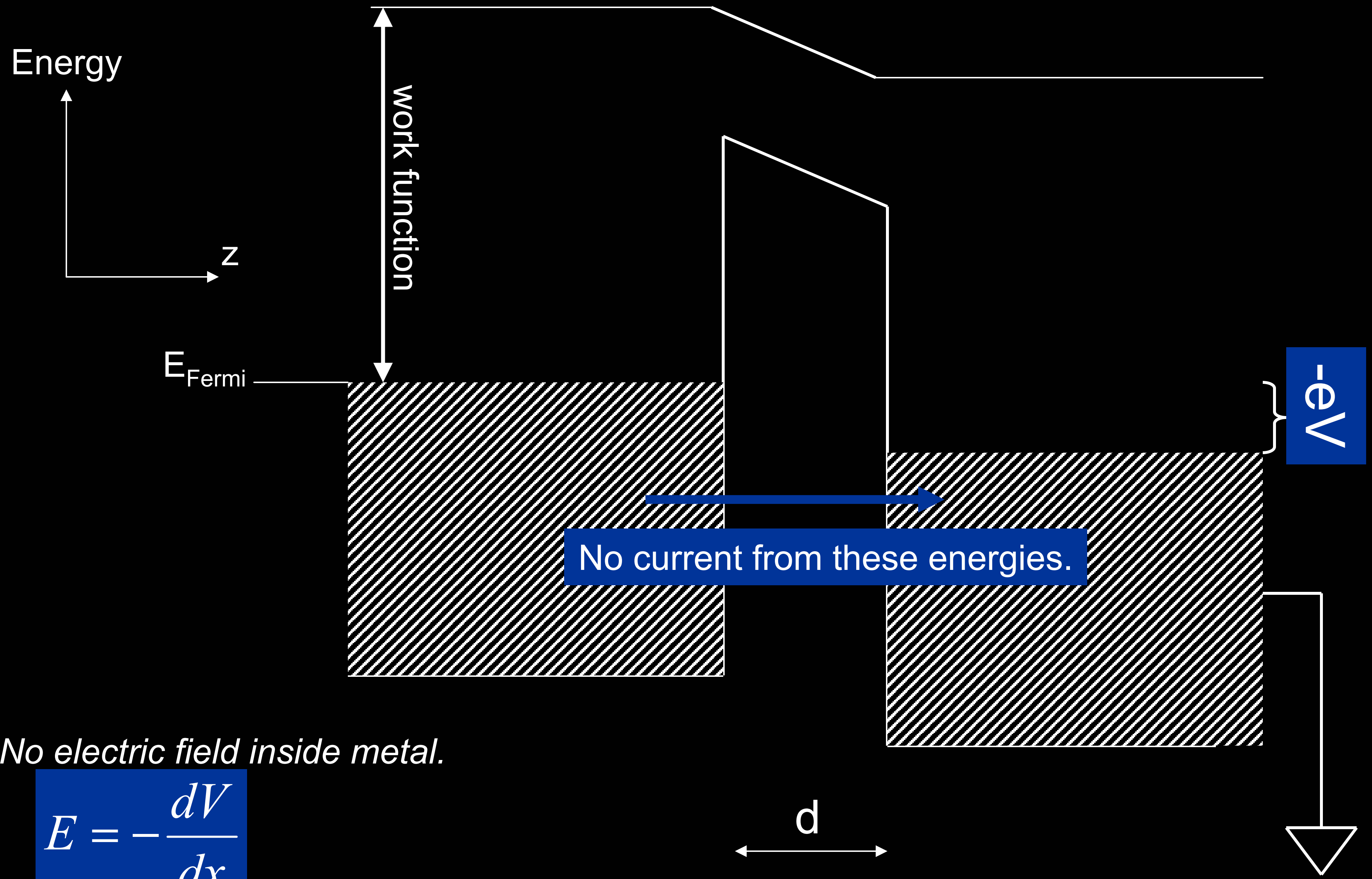

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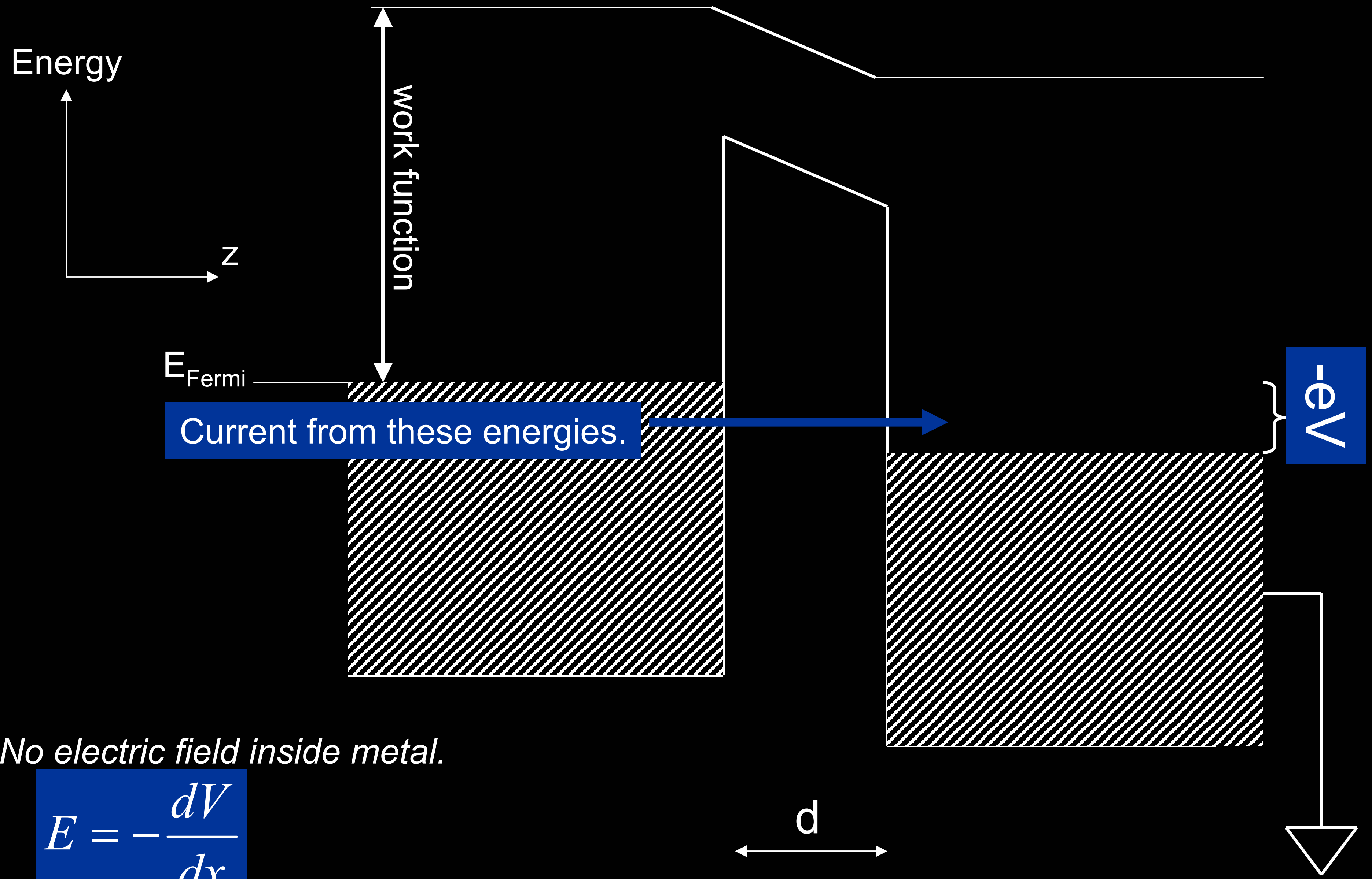

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$$E = -\frac{dV}{dx}$$

I-V curve

$$I = e\left(\left.\frac{\#\,electrons}{\sec ond}\right|_{R-L} - \left.\frac{\#\,electrons}{\sec ond}\right|_{L-R} \right)$$

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$$\left.\frac{\#\,electrons}{\sec ond}\right|_{L-R} = \sum_{leftelectronstates}\ \sum_{rightelectronstates} \left(\Pr ob_{leftelectronstateoccupied}\right)\left(\Pr ob_{rightelectronstateempty}\right)T$$

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Treat particles in left as “particle in a box”
Recall our way of labeling states, and each state has energy:

$$E = \frac{\hbar^2 (\pi / L)^2}{2m}({n_x}^2 + {n_y}^2 + {n_z}^2)$$

I-V curve

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$$\rightarrow \sum_{n_x, n_y, n_z} \sum_{m_x, m_y, m_z} \left(\mathrm{P}_{n_x, n_y, n_z}\right)\left(1 - \mathrm{P}_{m_x, m_y, m_z}\right) T$$

I-V curve

$$\left.\frac{\#\,electrons}{\sec ond}\right|_{L-R} = \sum_{n_x,n_y,n_z}\sum_{m_x,m_y,m_z}\left(\mathrm{P}_{n_x,n_y,n_z}\right)\left(1-\mathrm{P}_{m_x,m_y,m_z}\right)T$$

Energy and momentum are conserved in physics so:

I-V curve

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Energy and momentum are conserved in physics so:

$$T = 0 \text{ unless}$$

$$n_x = m_x$$

$$n_y = m_y$$

$$E_{left} - eV = E_{right}$$

I-V curve

$$\left.\frac{\#\,electrons}{second}\right|_{L-R} = \sum_{n_x,n_y,n_z}\sum_{m_x,m_y,m_z}\left(\mathrm{P}_{n_x,n_y,n_z}\right)\left(1-\mathrm{P}_{m_x,m_y,m_z}\right)T$$

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$$\Rightarrow \frac{\hbar^2(\pi/L)^2}{2m}\left(n_x^{\ 2}+n_y^{\ 2}+n_z^{\ 2}\right) - eV = \frac{\hbar^2(\pi/L)^2}{2m}\left(m_x^{\ 2}+m_y^{\ 2}+m_z^{\ 2}\right)$$
$$\Rightarrow \frac{\hbar^2(\pi/L)^2}{2m}n_z^{\ 2} - eV = \frac{\hbar^2(\pi/L)^2}{2m}m_z^{\ 2}$$

I-V curve

$$\left.\frac{\#\,electrons}{\sec ond}\right|_{L-R} = \sum_{n_x,n_y,n_z}\ \sum_{m_x,m_y,m_z} \left(\mathrm{P}_{n_x,n_y,n_z}\right)\left(1-\mathrm{P}_{m_x,m_y,m_z}\right)T$$

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$$\mathrm{P}_{n_x,n_y,n_z} = \frac{1}{1-e^{\left(\frac{\hbar^2(\pi/L)^2}{2m}\left(n_x{}^2+n_y{}^2+n_z{}^2\right)-E_f\right)/kT}} = f(E_L)$$

I-V curve

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I-V curve

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I-V curve

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$$\left.\frac{\#\,electrons}{second}\right|_{L-R} \rightarrow \sum_{n_x,n_y,n_z}\left(f(E_L)\right)\left(1-f(E_L+eV)\right)T$$

I-V curve

$$\left.\frac{\#\,electrons}{\sec ond}\right|_{L-R} \rightarrow \sum_{n_x, n_y, n_z} \left(f(E_L) \right)\left(1 - f(E_L + eV) \right) T$$

I-V curve

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A similar calculation shows:

$$\left.\frac{\#\,electrons}{second}\right|_{R-L} \rightarrow \sum_{n_x,n_y,n_z} \left(f(E_L+eV)\right)\left(1-f(E_L)\right)T$$

I-V curve

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Since:

$$I = e\left(\left.\frac{\#\,electrons}{\sec ond}\right|_{R-L} - \left.\frac{\#\,electrons}{\sec ond}\right|_{L-R}\right)$$

I-V curve

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Since:

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We have:

$$I = e\sum_{n_x,n_y,n_z} \left[\left(f(E_L) - f(E_L+eV)\right)\right]T$$

A nice, simple result.

I-V curve

$$I = e \sum_{n_x, n_y, n_z} \left[\left(f(E_L) - f(E_L + eV) \right) \right] T$$

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In the macro world, states are very finely spaced and we have (discuss):
(Later in the class we will see that this fails in nanosized circuits.)

$$\sum_{n_x} \rightarrow \int dn_x \qquad \sum_{n_y} \rightarrow \int dn_y \qquad \sum_{n_z} \rightarrow \int dn_z$$

I-V curve

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$$\sum_{n_x} \rightarrow \int dn_x \qquad \sum_{n_y} \rightarrow \int dn_y \qquad \sum_{n_z} \rightarrow \int dn_z$$

$$I \rightarrow e \int dn_x \int dn_y \int dn_z \left[\left(f(E_L) - f(E_L + eV) \right) \right] T$$

I-V curve

$$I \to e\int dn_x \int dn_y \int dn_z \left[\left(f(E_L) - f(E_L + eV)\right)\right] T$$

I-V curve

$$I \to e\int dn_x \int dn_y \int dn_z \left[\left(f(E_L) - f(E_L + eV)\right)\right]T$$

$$I \to e\int dn_x \int dn_y \int \frac{m}{\hbar^2(\pi/L)^2} \frac{1}{\sqrt{E_L - \frac{\hbar^2(\pi/L)^2}{2m}\left(n_x{}^2 + n_y{}^2\right)}} dE_L \left[\left(f(E_L) - f(E_L + eV)\right)\right]T$$

I-V curve

$$I \to e\int dn_x \int dn_y \int dn_z \left[\left(f(E_L) - f(E_L + eV)\right)\right] T$$

$$I \to e\int dn_x \int dn_y \int \frac{m}{\hbar^2(\pi/L)^2} \frac{1}{\sqrt{E_L - \frac{\hbar^2(\pi/L)^2}{2m}\left(n_x{}^2 + n_y{}^2\right)}} dE_L \left[\left(f(E_L) - f(E_L + eV)\right)\right] T$$

$$I \approx e\int dn_x \int dn_y \frac{m}{\hbar^2(\pi/L)^2} \frac{1}{\sqrt{E_F - \frac{\hbar^2(\pi/L)^2}{2m}\left(n_x{}^2 + n_y{}^2\right)}} T \int dE_L \left[\left(f(E_L) - f(E_L + eV)\right)\right]$$

I-V curve

$$I \to e\int dn_x \int dn_y \int dn_z \left[\left(f(E_L) - f(E_L + eV)\right)\right] T$$

$$I \to e\int dn_x \int dn_y \int \frac{m}{\hbar^2(\pi/L)^2} \frac{1}{\sqrt{E_L - \frac{\hbar^2(\pi/L)^2}{2m}\left(n_x{}^2 + n_y{}^2\right)}} dE_L \left[\left(f(E_L) - f(E_L + eV)\right)\right] T$$

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$$\int dE_L \left[\left(f(E_L) - f(E_L + eV)\right)\right] \approx eV$$

(show on board)

I-V curve

$$I \to e\int dn_x \int dn_y \int dn_z \left[\left(f(E_L) - f(E_L + eV)\right)\right]T$$

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$$I \approx e\int dn_x \int dn_y \frac{m}{\hbar^2(\pi/L)^2} \frac{1}{\sqrt{E_F - \frac{\hbar^2(\pi/L)^2}{2m}\left(n_x^{\,2} + n_y^{\,2}\right)}} T\int dE_L \left[\left(f(E_L) - f(E_L + eV)\right)\right]$$

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(show on board)

$$I \approx (eV)eT \frac{m}{\hbar^2(\pi/L)^2} \int_0^\infty dn_x \int_0^\infty dn_y \frac{1}{\sqrt{E_F - \frac{\hbar^2(\pi/L)^2}{2m}\left(n_x^{\,2} + n_y^{\,2}\right)}}$$

I-V curve

$$I \to e\int dn_x \int dn_y \int dn_z \left[\left(f(E_L) - f(E_L + eV)\right)\right]T$$

$$I \to e\int dn_x \int dn_y \int \frac{m}{\hbar^2(\pi/L)^2} \frac{1}{\sqrt{E_L - \frac{\hbar^2(\pi/L)^2}{2m}\left(n_x{}^2 + n_y{}^2\right)}} dE_L \left[\left(f(E_L) - f(E_L + eV)\right)\right]T$$

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(show on board)

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$$I \approx (eV)(\text{constant})$$