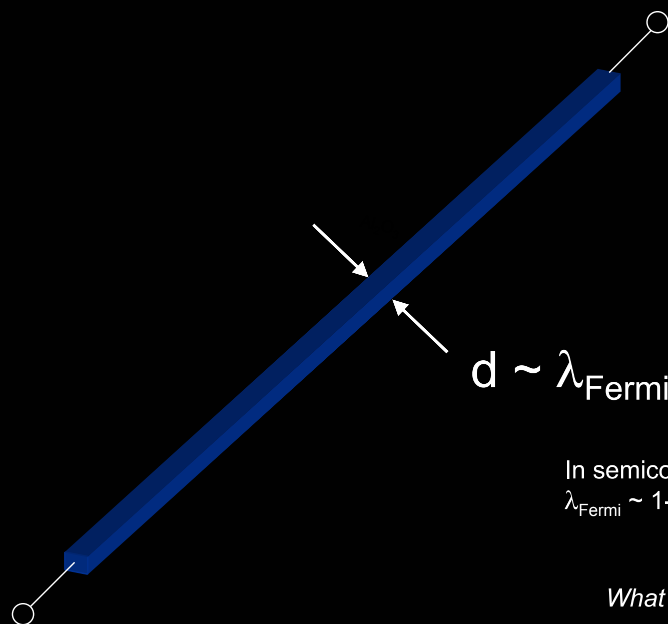
Lecture 10: Nanowires



In semiconductors,
λ_{Fermi} ~ 1-10 nm

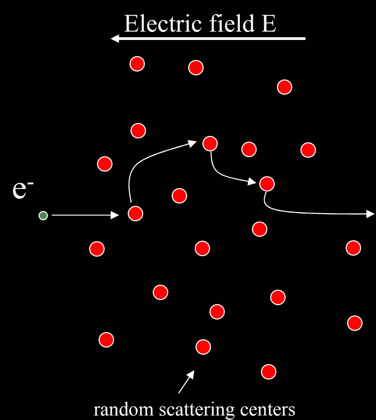
What is the resistance?

Drift current

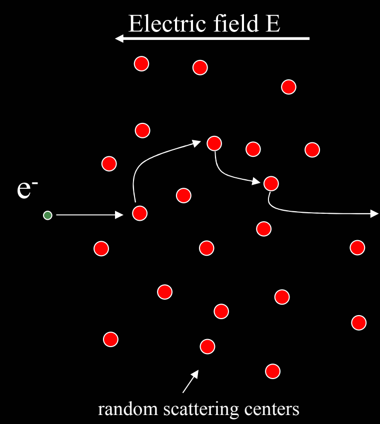
- Caused by electric field
- Electron density constant
- Analogy: swarm of mosquitoes in the wind

Drift: Drude model

$$F = ma$$



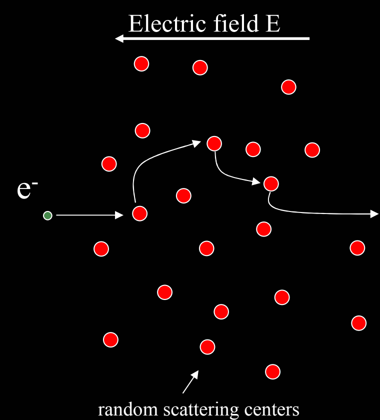
Drift: Drude model



$$F = ma$$

$$eE = m\frac{\partial v}{\partial t}$$

Drift: Drude model

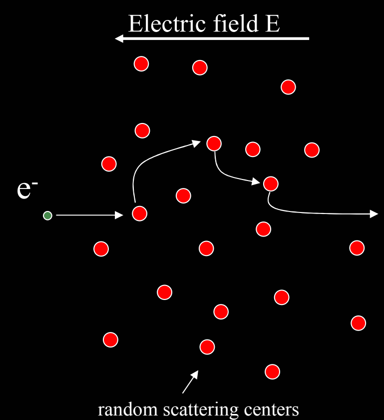


$$F = ma$$

$$eE = m \frac{\partial v}{\partial t}$$

$$v_{avg} = \frac{e\tau}{m} E$$

Drift: Drude model

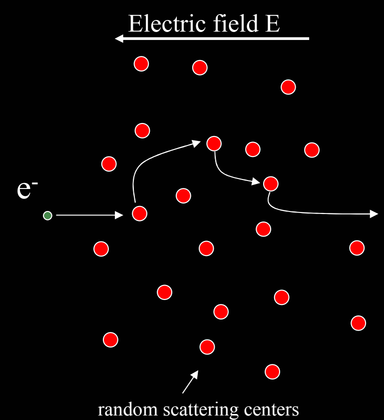


$$F = ma$$

$$eE = m\frac{\partial v}{\partial t}$$

$$v_{avg} = \underbrace{\frac{e\tau}{m}}_{\mu} E$$

Drift: Drude model



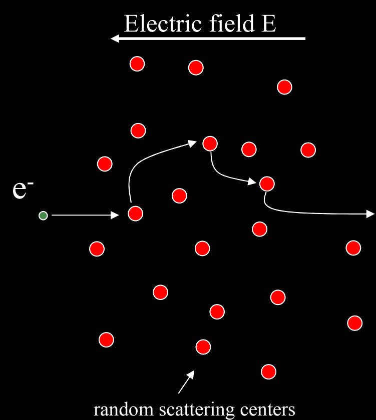
$$F = ma$$

$$eE = m \frac{\partial v}{\partial t}$$

$$v_{avg} = \underbrace{\frac{e\tau}{m}}_{\mu} E$$

$$j = ne\, v_{avg} = \frac{ne^2\tau}{m} E$$

Drift: Drude model



$$F = ma$$

$$eE = m \frac{\partial v}{\partial t}$$

$$v_{avg} = \underbrace{\frac{e\tau}{m}}_{\mu} E$$

$$j = ne\, v_{avg} = \underbrace{\frac{ne^2\tau}{m}}_{\sigma} E$$

Types of scattering

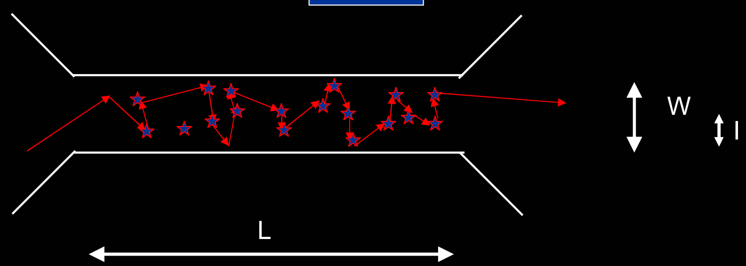
- Electron-phonon:
  - Very temperature dependent
  - Phonons are lattice vibrations
  - At low temperatures, lattice is “perfectly still”
- Impurity scattering
  - Temperature independent
  - Depends on impurity concentration

$$\frac{1}{\tau_{total}} = \frac{1}{\tau_{electron-phonon}} + \frac{1}{\tau_{impurity}}$$

Ballistic vs. diffusive transport

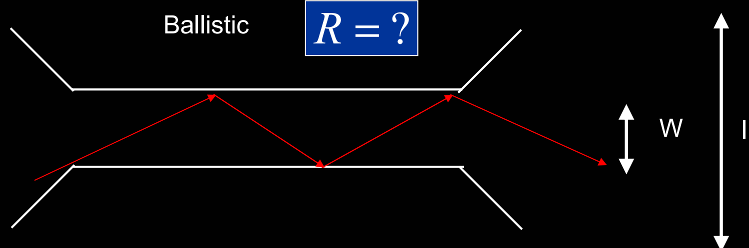
Diffusive

$$R = \frac{L}{W^2}\rho$$

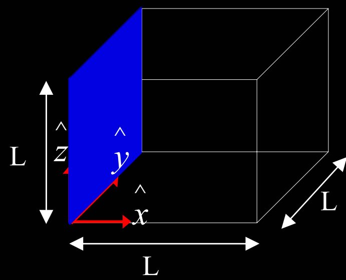


Ballistic

$$R = ?$$



Particle in a box:



$$\psi(\vec{r}) = (2i)^3 A \cdot \sin(k_{n_x} x) \cdot \sin(k_{n_y} y) \cdot \sin(k_{n_z} z)$$

$$k_{n_x} = n_x \pi / L$$

$$k_{n_y} = n_y \pi / L$$

$$k_{n_z} = n_z \pi / L$$

$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2 (\pi / L)^2}{2m} (n_x{}^2 + n_y{}^2 + n_z{}^2)$$

These are the allowed energy levels, or "quantum states"

Particle in a nanowire:

$$\psi(\vec{r}) = (2i)^3 A \cdot \sin(k_{n_x} x) \cdot \sin(k_{n_y} y) \cdot \sin(k_{n_z} z)$$

$$k_{n_x} = n_x \pi / L_x$$

$$k_{n_y} = n_y \pi / L_y$$

$$k_{n_z} = n_z \pi / L_z$$

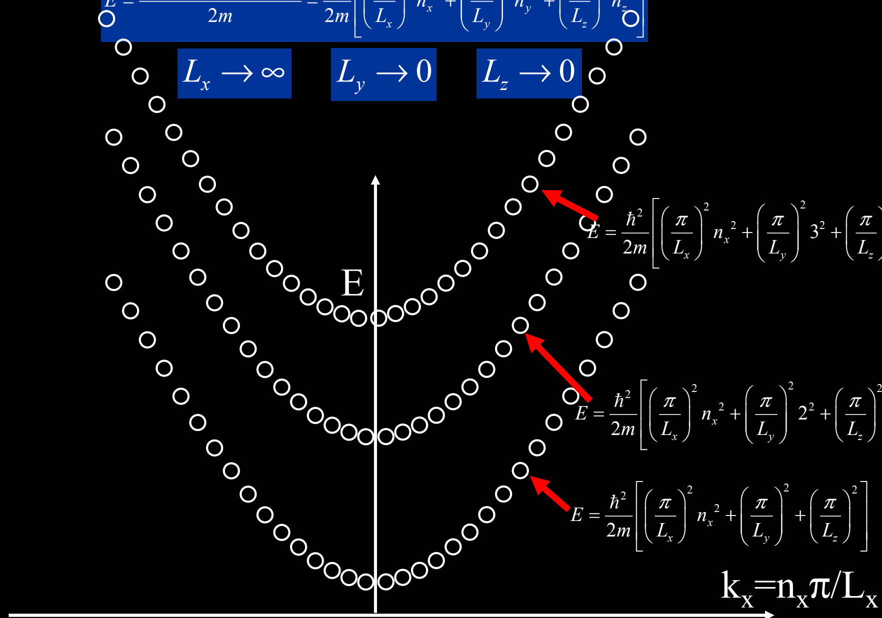
$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{\pi}{L_z}\right)^2 n_z{}^2\right]$$

These are the allowed energy levels, or “quantum states”

Limits:

$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{\pi}{L_z}\right)^2 n_z{}^2\right]$$

$L_x \rightarrow \infty$ $\quad L_y \rightarrow 0$ $\quad L_z \rightarrow 0$



$$E = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 3^2 + \left(\frac{\pi}{L_z}\right)^2\right]$$

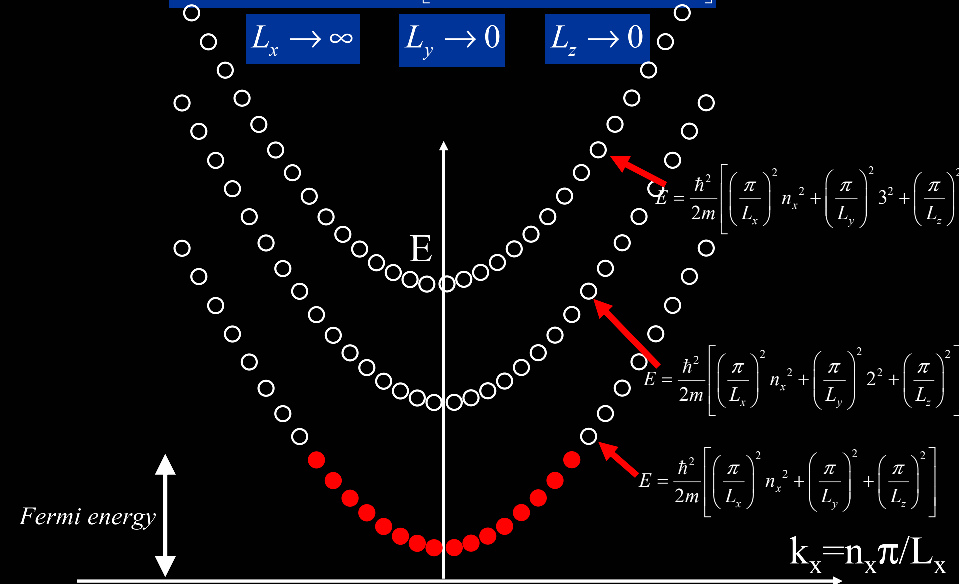
$$E = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 2^2 + \left(\frac{\pi}{L_z}\right)^2\right]$$

$$E = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 + \left(\frac{\pi}{L_z}\right)^2\right]$$

1d system:

$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{\pi}{L_z}\right)^2 n_z{}^2\right]$$

$L_x \rightarrow \infty$ $\quad L_y \rightarrow 0$ $\quad L_z \rightarrow 0$



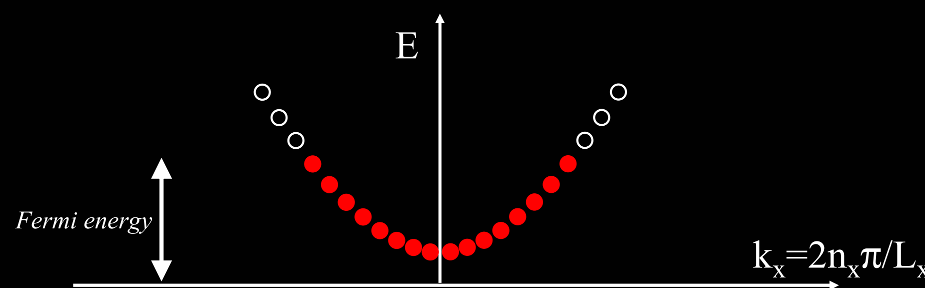
Positive and negative k-vectors:

Particle in a box: (positive k-vectors only)

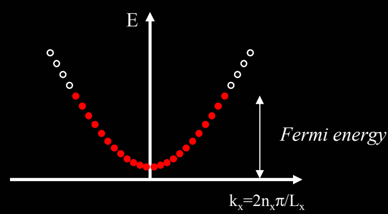
$$E = \frac{\hbar^2 (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)}{2m} = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{\pi}{L_z}\right)^2 n_z{}^2\right]$$

"Born-Von Karman" boundary conditions: (positive *and* negative k-vectors)

$$E = \frac{\hbar^2}{2m}\left[\left(\frac{2\pi}{L_x}\right)^2 n_x{}^2 + \left(\frac{2\pi}{L_y}\right)^2 n_y{}^2 + \left(\frac{2\pi}{L_z}\right)^2 n_z{}^2\right]$$



Single sub-band:



$$I = \frac{\text{charge}}{\text{time}} = e \cdot \frac{\#\text{elec}}{\text{time}} = e \cdot \text{v} \frac{\#\text{elec}}{\text{length}}$$

Different electrons have different velocities.

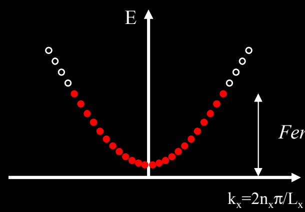
$$\text{v} = \frac{\text{momentum}}{\text{mass}} = \frac{\text{p}}{\text{m}} = \frac{\hbar k}{\text{m}}$$

$$I = I_{right\ goers} - I_{left\ goers}$$

$$I_{right\ goers} = \sum_{right\ going\ electrons} e\ \text{v} \frac{1}{\text{length}} = \sum_{\substack{occupied\ states \\ (right\ goers)}} e\ \text{v} \frac{1}{\text{length}}$$

$$I_{right\ goers} = \frac{e}{L_x} \sum_{k_x=0}^{k_F} \frac{\hbar k_x}{m} = \frac{e}{L_x} \sum_{n_x=0}^{n_F} \frac{\hbar \left(n_x 2\pi / L_x \right)}{m} = \frac{e 2\pi\hbar}{m L_x^{\ 2}} \sum_{n_x=0}^{n_F} n_x$$

Single sub-band:



$$I_{right\ goers} = \frac{e2\pi\hbar}{mL_x^{\ 2}} \sum_{n_x=0}^{n_F} n_x \rightarrow \frac{e2\pi\hbar}{mL_x^{\ 2}} \int_0^{n_F} n_x dn_x$$

Change of variables:

$$E = \frac{\hbar^2 k_x^{\ 2}}{2m} = \frac{\hbar^2 (2n_x \pi / L_x)^2}{2m} \Rightarrow dE = \frac{4\hbar^2 (\pi / L_x)^2}{m} n_x dn_x$$

$$\Rightarrow n_x dn_x = \frac{m}{4\hbar^2 (\pi / L_x)^2} dE$$

$$I_{right\ goers} = \frac{e\pi\hbar}{2mL_x^{\ 2}} \int_0^{n_F} n_x dn_x \rightarrow \frac{e\pi\hbar}{2mL_x^{\ 2}} \frac{m}{\hbar^2 (\pi / L_x)^2} \int dE = \frac{e}{h} \int dE$$

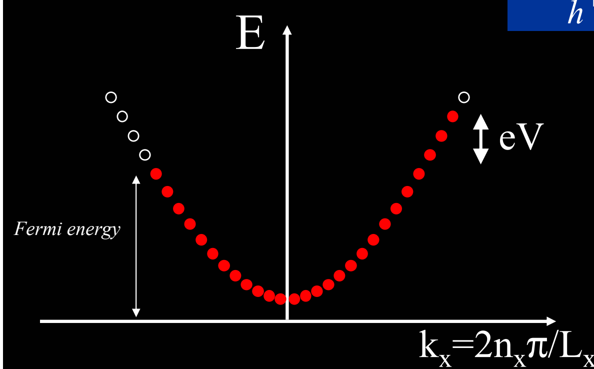
Resistance quantum

Ballistic conductor

$$I_{right\ goers} = \frac{e}{h}\int dE \qquad I_{right\ goers} = \frac{e}{h}\int dE$$

$$I = \frac{e}{h}\left[\int dE_{\text{right goers}} - \int dE_{\text{left goers}}\right]$$

$$I = \frac{e}{h}\left[(E_F + eV) - E_F\right] = \frac{e^2}{h}V$$



$$V = I\frac{h}{e^2} = IR_{quantum}$$

$$R_{quantum} = \frac{h}{e^2} = 25\ k\Omega$$

With spin:

$$R_{quantum} = \frac{h}{2e^2} = 12.5\ k\Omega$$