Lecture 9: Laser output power

- Reading: Verdeyen ch. 9
- Optional: Siegman ch. 12

Steady-state intensity:

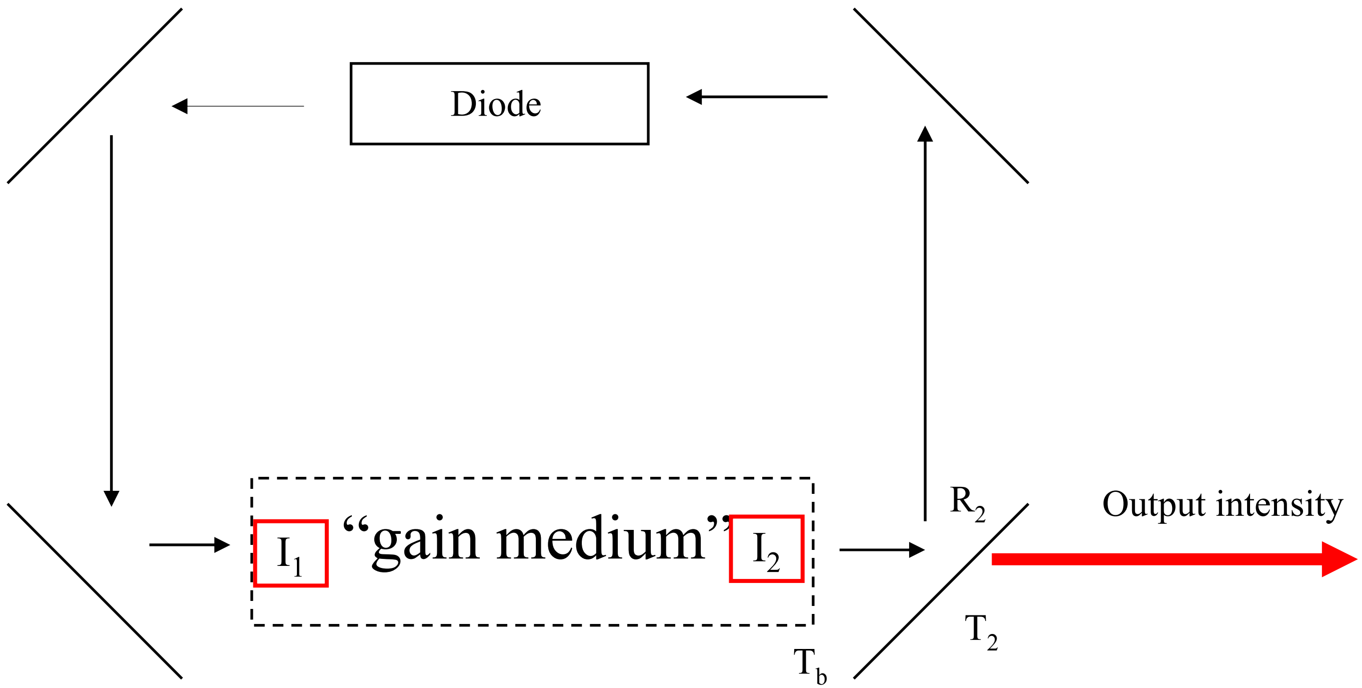
- Assume that we are not letting light out yet.
- There is only absorption in the cavity.
- What is intensity in the cavity?
- Gain is saturated at loss (otherwise intensity would increase with time)
- Loss is a property of the cavity
- Gain is function of intensity
- Find intensity that gives the gain=loss, and that is the internal intensity of cavity.
- I_s (function of lifetime and frequency) important scale of intensity to expect.

Output power

- Very reflective mirrors: Internal intensity high, but little of it gets out
- Very transmitting mirrors: Lots of the light gets out but internal intensity is low.
- Which is better? Need a quantitative theory.
- Ring laser
- Standing wave laser

Ring laser

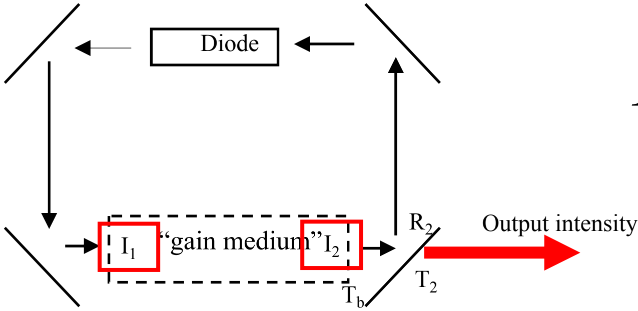
Easy to analyze because light only propagates in one direction.



$$I_2 \rightarrow I_2\, T_b \rightarrow I_2\, T_b\, R_2 \rightarrow I_2\, T_b\, R_2\, L = I_1 \rightarrow GI_1 = G(I_2\, T_b\, R_2\, L) = I_2$$

But we have a formula relating I_2 and I_1.

Ring laser



$$I_2 T_b R_2 L = I_1 \Rightarrow I_2 / I_1 = T_b R_2 L$$

$$\ln \frac{I_2}{I_1} + \frac{I_2 - I_1}{I_s} = \gamma_0 l_g$$

$$\Rightarrow \frac{I_2}{I_s} = \frac{\gamma_0 l_g - \ln(T_b R_2 L)}{1 - T_b R_2 L}$$

Threshold: round trip loss * gain = 1:

$$\Rightarrow T_b R_2 L \cdot e^{\gamma_{th} l_g} = 1 \Rightarrow \gamma_{th} = \frac{\ln(T_b R_2 L)}{l_g}$$

$$\Rightarrow \frac{I_2}{I_s} = \frac{\gamma_0 l_g - \gamma_{th} l_g}{1 - T_b R_2 L}$$

$$\Rightarrow I_{out} = I_2 \cdot T_b \cdot T_2 = T_b \cdot T_2 \cdot I_s \frac{\gamma_0 l_g - \gamma_{th} l_g}{1 - T_b R_2 L}$$

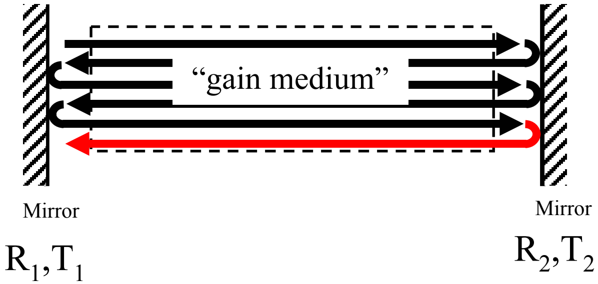
So we now know the output power!

Pause and reflect:

- Started with plane waves
- Add mirrors
- Stimulated emission/absorption → gain
- Optical reflectivities, losses, etc.
- All we need are lifetimes and energies, and we can predict the laser output power!
- Is it true? (I hope so!)

- Verdeyen goes on to discuss limits of highly reflective mirrors and highly transmitting mirrors for ring lasers.
- We will skip and go right to standing wave lasers.

Standing wave laser:



We have waves going in both directions.
The intensity of each is $I_+(z)$ and $I_-(z)$

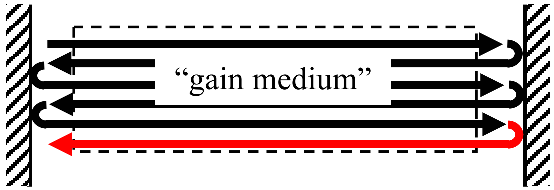
α_{int}, α_{ext}: loss per unit length
(discuss on board, put in to be consistent with Verdeyen)
Rs, Ts a little different from Verdeyen. Forget α_{ext}

$$\frac{\partial I_+}{\partial z} = \gamma(z) I_+ \qquad \frac{\partial I_-}{\partial z} = -\gamma(z) I_-$$

$$\gamma = \frac{\gamma_0}{1 + I / I_s} \rightarrow \gamma(z) = \frac{\gamma_0}{1 + I(z) / I_s} = \frac{\gamma_0}{1 + \left(I_+(z) + I_-(z)\right) / I_s}$$

Must solve for I!

High Q approximation



R_1, T_1 R_2, T_2

α_{int}: loss per unit length

"High Q approximation:"
$R_1 \approx 1 \quad R_2 \approx 1$

Then intensity builds up to a high value internally.
Gain medium goes into saturation.
So we approximate $I_+(z)$, $I_-(z)$ as constant: $\equiv I_{circ}$

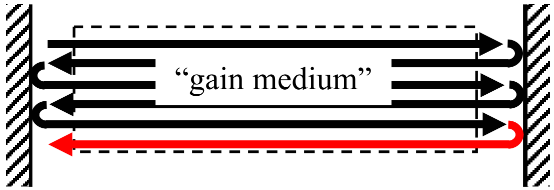
$$\gamma(z) = \frac{\gamma_0}{1 + \left(I_+(z) + I_-(z)\right)/I_s} = \frac{\gamma_0}{1 + \left(I_{circ} + I_{circ}\right)/I_s} = \frac{\gamma_0}{1 + 2I_{circ}/I_s}$$

$$\frac{\partial I_+}{\partial z} = \gamma(z) I_+ = \frac{\gamma_0}{1 + 2I_{circ}/I_s} I_+$$

$$\Rightarrow \text{One way gain} = e^{\frac{\gamma_0}{1+2I_{circ}/I_s} l_g}$$

(Not entirely self-consistent.)

Standing wave laser (high Q):



R_1, T_1 R_2, T_2

α_{int}: loss per unit length

$$\text{One way gain} = e^{\frac{\gamma_0}{1+2I_{circ}/I_s} l_g}$$

$$\text{Two way gain} = e^{\frac{2\gamma_0}{1+2I_{circ}/I_s} l_g}$$

$$\text{Two way gain w/loss} = e^{\frac{2\gamma_0}{1+2I_{circ}/I_s} l_g - 2\alpha_{\text{int}} l_g}$$

(Discuss on board)

Loss and gain must be equal for steady state:

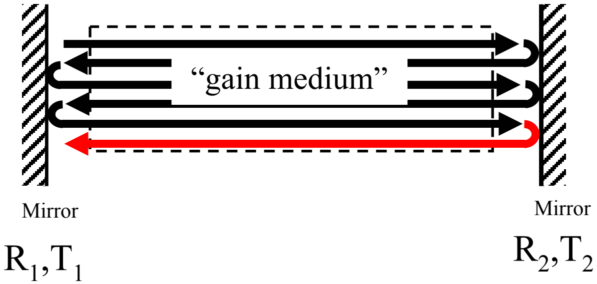
$$e^{\frac{2\gamma_0}{1+2I_{circ}/I_s} l_g} e^{-2\alpha_{\text{int}} l_g} = R_1 R_2$$

$$\ln(xy) = \ln(x) + \ln(y)$$

$$\frac{2\gamma_0}{1+2I_{circ}/I_s} l_g - 2\alpha_{\text{int}} l_g = \ln(R_1) + \ln(R_2)$$

$$I_{circ} = \frac{1}{2} I_s \left[\frac{\gamma_0 l_g}{\alpha_{\text{int}} l_g + \ln(1/R_2) + \ln(1/R_1)} - 1 \right]$$

Output power (high Q):

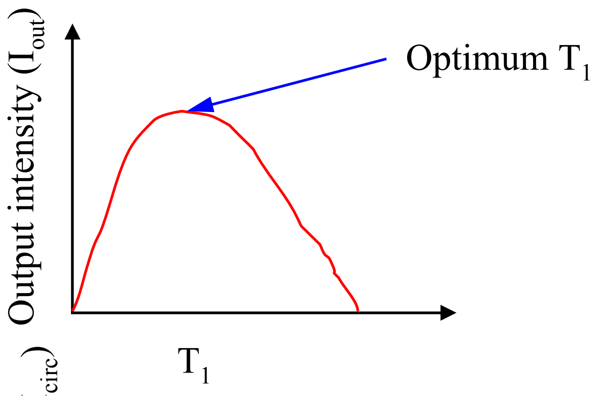


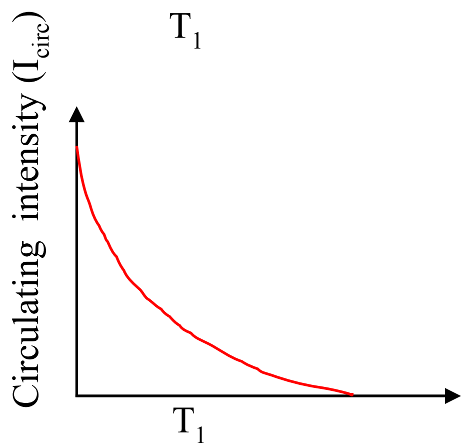
$$I_{output} = T_1 \cdot \frac{1}{2} I_s \left[\frac{\gamma_0 l_g}{\alpha_{\text{int}} l_g + T_1} - 1 \right]$$

$$I_{circ} = \frac{1}{2} I_s \left[\frac{\gamma_0 l_g}{\alpha_{\text{int}} l_g + T_1} - 1 \right]$$

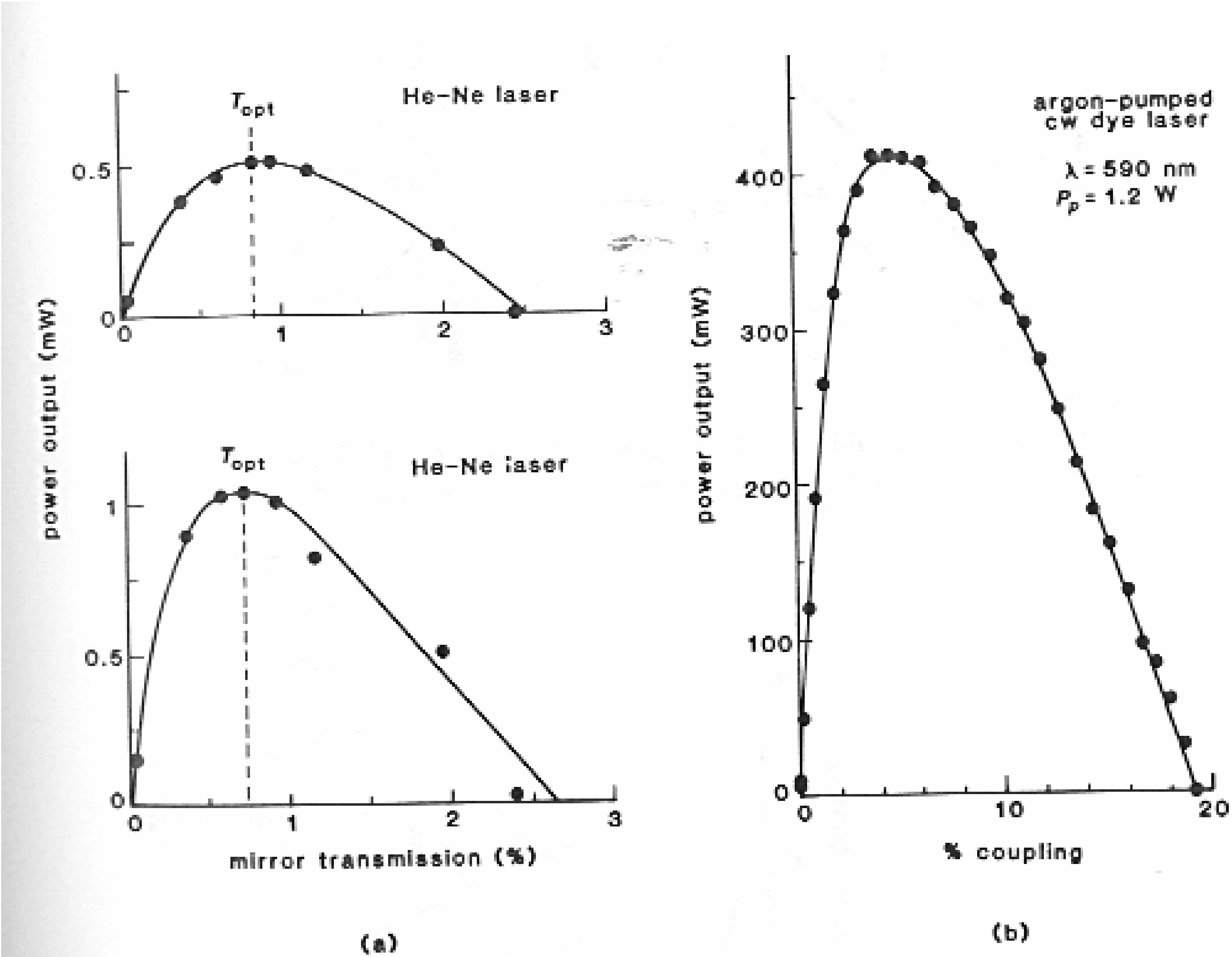
α_{int}, α_{ext}: loss per unit length

Now let's ask: What value of R_1 gives the largest I_{output}?



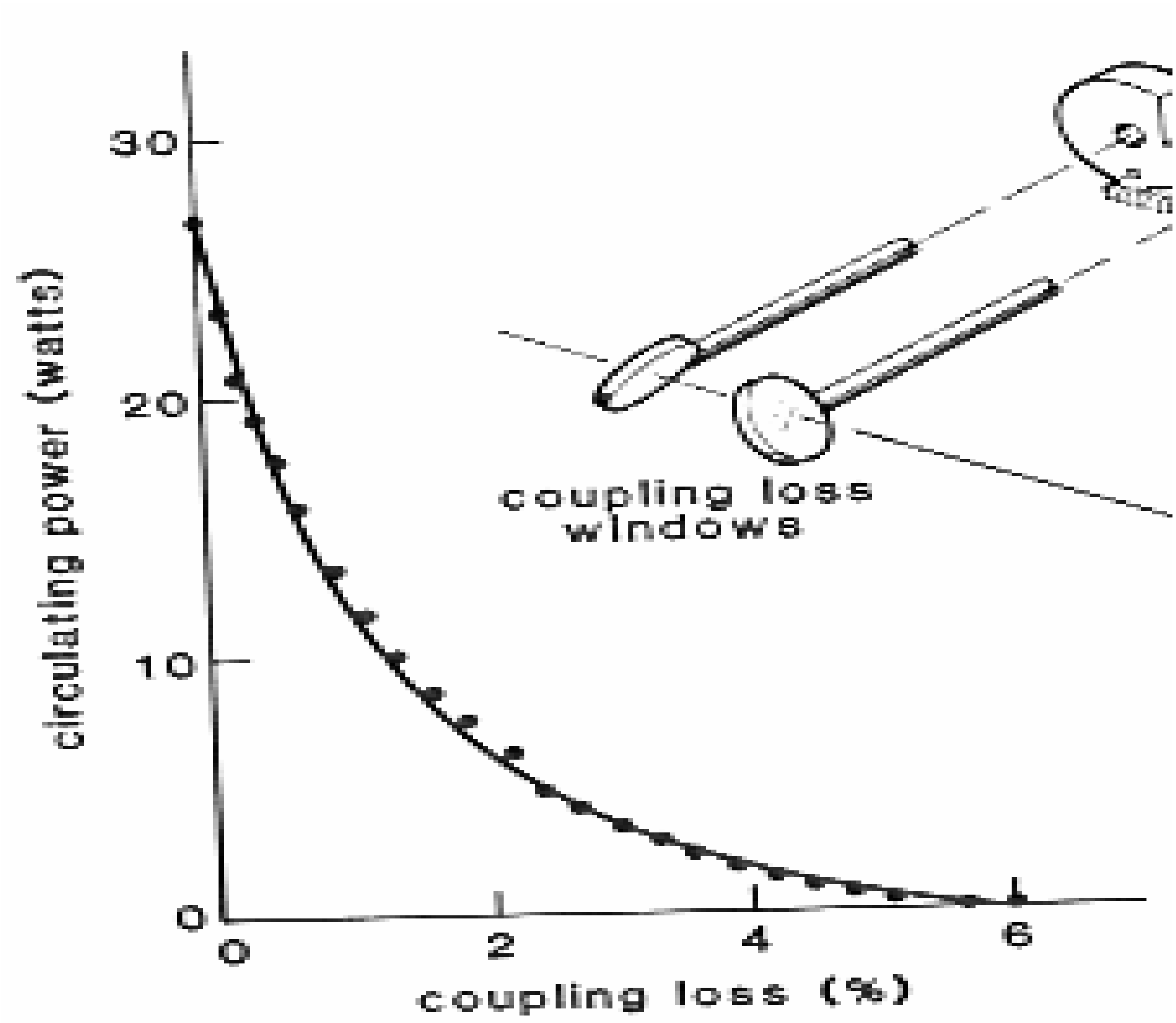


Is it true?



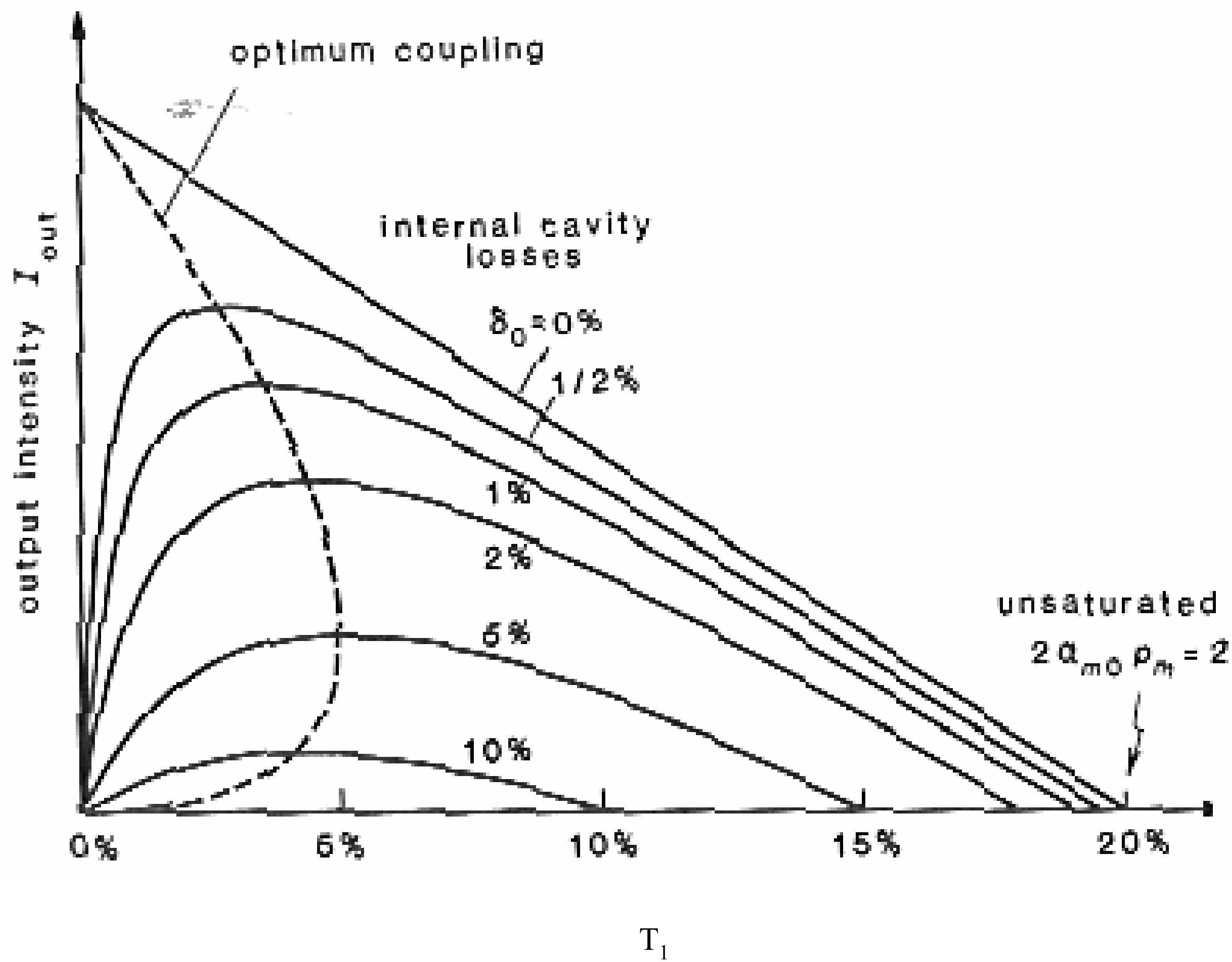
From Siegman, Lasers

Is it true?



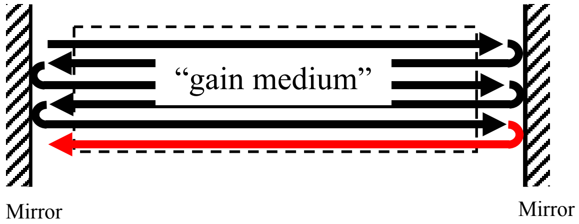
From Siegman, Lasers

Effect of internal losses:



From Siegman, Lasers

Rigrod analysis:



R_1, T_1 R_2, T_2

α_{int}, loss per unit length

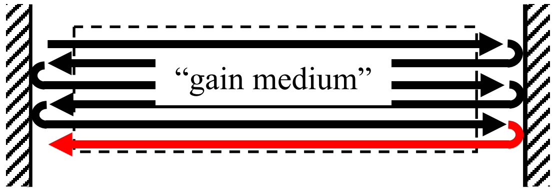
$$\frac{\partial I_+}{\partial z} = \gamma(z) I_+$$

$$\frac{\partial I_-}{\partial z} = -\gamma(z) I_-$$

$$\frac{\partial}{\partial z}\left(I_+ \cdot I_-\right) = \frac{\partial I_+}{\partial z} \cdot \left(I_-\right) + \frac{\partial I_-}{\partial z} \cdot \left(I_+\right) = \gamma(z) I_+ \cdot \left(I_-\right) + -\gamma(z) I_- \cdot \left(I_+\right) = 0$$

Reference: W.W. Rigrod, "Saturation effects in high-gain laser", *Journal of Applied Physics* **36**, 2487-2490 (August, 1965).

Rigrod analysis:



R_1, T_1 R_2, T_2

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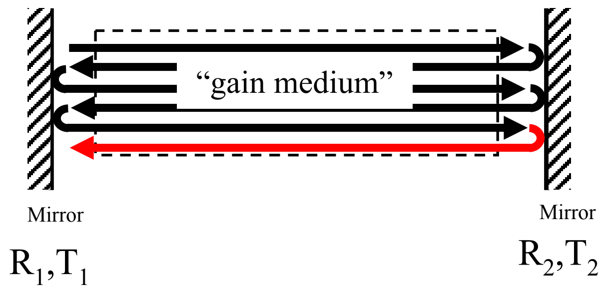
$$\Rightarrow \left(I_+ \cdot I_-\right) = \text{a constant!} \equiv \text{C}$$

Reference: W.W. Rigrod, "Saturation effects in high-gain laser", *Journal of Applied Physics* **36**, 2487-2490 (August, 1965).

Standing wave laser:



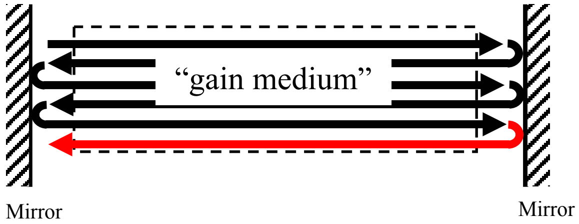
$$\gamma(z) = \frac{\gamma_0}{1 + \left(I_+(z) + I_-(z)\right)/I_s} = \frac{\gamma_0}{1 + \left(I_+(z) + C/I_+(z)\right)\Big/ I_s}$$

$$\frac{\partial I_+}{\partial z} = \gamma(z) I_+ \Rightarrow$$

$$\frac{\partial I_+}{\partial z} = \frac{\gamma_0}{1 + \left(I_+(z) + C/I_+(z)\right)\Big/ I_s} I_+$$

Lets integrate and solve for I_+

Standing wave laser:



R_1,T_1 R_2,T_2

$$\frac{\partial I_+}{\partial z} = \frac{\gamma_0}{1 + \left(I_+(z) + C/I_+(z)\right) \big/ I_s} I_+$$

$$\partial I_+ \frac{1}{I_s} + \frac{1}{I_+} + + \frac{C}{I_+{}^2 I_s} = \gamma_0 \partial z$$

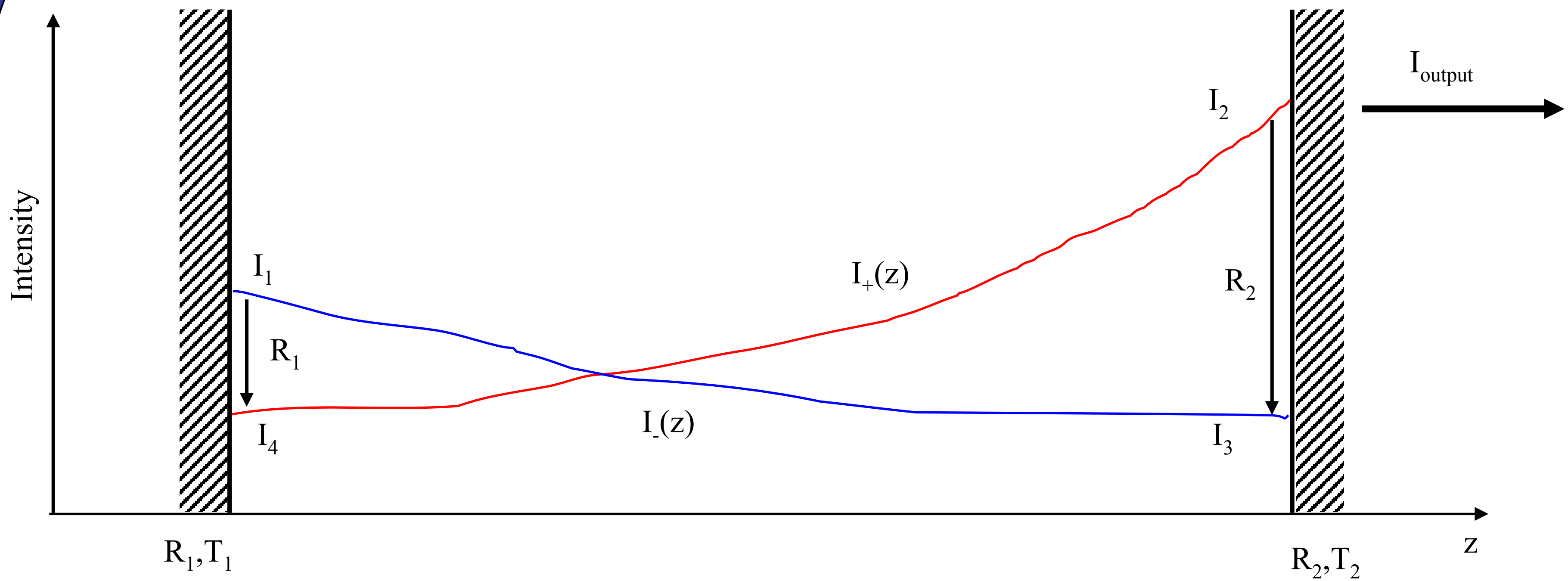
$$\int_{I_1}^{I_2} \partial I_+ \left(\frac{1}{I_s} + \frac{1}{I_+} + + \frac{C}{I_+{}^2 I_s} \right) = \int_0^{l_g} \gamma_0 \partial z$$

$$\frac{I_2 - I_1}{I_s} + \ln\left(\frac{I_2}{I_1}\right) - \frac{C}{I_s}\left(\frac{1}{I_2} - \frac{1}{I_1}\right) = l_g \gamma_0$$

Do same thing for I_- in terms of I_3, I_4.

What are limits of integration?

Integration limits:

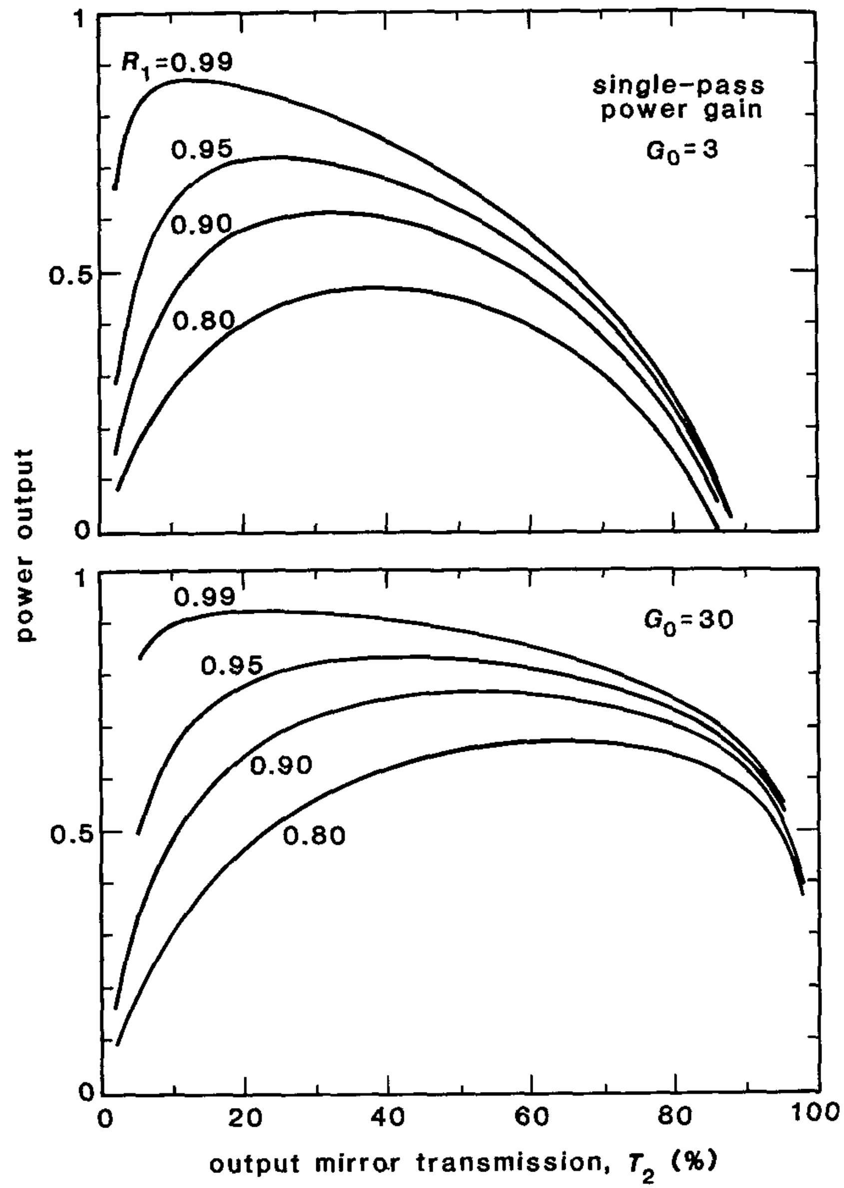


If you want the details, see Rigrod's paper.
Using these boundary conditions you can solve for the constant C and hence I_+, I_-, and I_{output}

$$I_{output} = T_2 \cdot I_2 = T_2 \cdot I_s \left[\frac{\gamma_0 l_g - 0.5 \cdot \ln(R_1 R_2)}{\left(1 - \sqrt{R_1 R_2}\right)\left(1 + \sqrt{R_2 / R_1}\right)} \right]$$

This covers all cases. Must be evaluated numerically for typical examples.

Typical example:



From Siegman, Lasers

That concludes our discussion of laser oscillation in the steady state.

On the horizon

- Time-dependent laser oscillation (Q-switching, mode-locking)
- Example laser systems
- Beams and propagation in fibers
- Semiconductor lasers