ECE 275B
Midterm
Monday, Feb. 10 2002

1A	1B	1C	1D	1E	1F	2A	2B	3A	3B	3C	3D	Total
/5	/5	/5	/5	/5	/5	/20	/20	/15	/5	/5	/5	/100

1) In a three-level system, the energies are E_0, E_1, E_2.
Assume the system is in equilibrium with a reservoir of temperature T.
Let P_0 = the probability of being in the state with energy E_0 (varying from 0 to 1).
Similarly for P_1, P_2.

A) (5 points) What is the ratio P_0/P_1 in terms of T, E_0, E_1, E_2?

$$\frac{P_0}{P_1} = e^{(E_1-E_0)/kT}$$

B) (5 points) What is the ratio P_2/P_1 in terms of T, E_0, E_1, E_2?

$$\frac{P_2}{P_1} = e^{-(E_2-E_1)/kT}$$

C) (5 points) Numerically, what is $P_0+P_1+P_2$?

One

D) (5 points) Work out exactly what P_0 is in terms of T, E_0, E_1, E_2?

$$P_0 = \frac{e^{-E_0/kT}}{e^{-E_2/kT} + e^{-E_1/kT} + e^{-E_0/kT}}$$

E) (5 points) Work out exactly what P_1 is in terms of T, E_0, E_1, E_2?

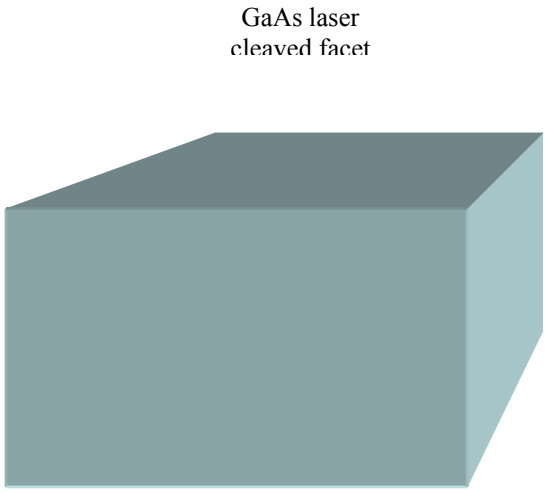
$$P_0 = \frac{e^{-E_1/kT}}{e^{-E_2/kT} + e^{-E_1/kT} + e^{-E_0/kT}}$$

F) (5 points) Work out exactly what P_3 is in terms of T, E_0, E_1, E_2?

$$P_0 = \frac{e^{-E_2/kT}}{e^{-E_2/kT} + e^{-E_1/kT} + e^{-E_0/kT}}$$

2) Consider a GaAs laser. Assume that the GaAs is the gain medium. The mirror is the cleaved facet. n=3.5.

A) (20 points) If the only loss is due to the finite reflectivity of the end of the laser, what does the one-way gain of the "gain medium" need to be to sustain lasing? The ends reflect because of the index of refraction mismatch between the GaAs and vacuum. Express your answer in dB.



The reflection coefficient is given by the formula in an above problem:

$$R \equiv \frac{I_r}{I_i} = \frac{E_r^{\,2}}{E_i^{\,2}} = r^2 \qquad r \equiv \frac{E_r}{E_i} = \frac{n_1 - n_2}{n_1 + n_2}$$

So R=0.308. So the gain most be equal to 1/0.3 = 3.24 in linear units. In dB, this is 5.1 dB.

Grading criteria:
15 points if you calculated 0.303 or 3.24 but did not convert to dB.
10 points if you wrote down correct loss formula but did not evaulate.
5 points if you said gain must exceed loss.
0 else.

B) (20 points) If the length of the gain medium is 1 mm, then what does the gain constant γ need to be to sustain lasing? Express your answer in cm^{-1}.

$$G = e^{\gamma l} \Rightarrow \gamma = \ln(G)/l = \ln(3.24)/0.1cm = 11.75cm^{-1}$$

Grading criteria:
10 points for writing down the above formula without evaulating numerically.
0 else.

3) Three-level system with energy E_0, E_1, E_2. The total number of atoms it N_{total}. N_0 atoms are in state with energy E_0. Similarly for N_1, N_2.

A) (15 points) In the absence of any external radiation, write down the rate equations for this system, i.e. dN_2/dt= … then dN_1/dt=… then dN_0/dt=… (In other words, neglect stimulated emission and absorption.) Use the notation defined in class for either the A coefficients or the lifetimes.

$$\frac{dN_2}{dt} = -A_{21}N_2 - A_{20}N_2$$
$$\frac{dN_1}{dt} = +A_{21}N_2 - A_{10}N_1$$
$$\frac{dN_0}{dt} = +A_{21}N_2 + A_{10}N_1 = N_{total} - \frac{dN_2}{dt} - \frac{dN_1}{dt}$$

B) (5 points) Solve for $N_0(t)$ with the initial condition that half the atoms are in the ground state (i.e. the state with energy E_0) and half the atoms are in state with energy E_1.

$$N_0(t) = N_{total}\left(1 - \frac{1}{2}e^{-A_{10}t}\right)$$

C) (5 points) Solve for $N_1(t)$ with the same initial condition as B.

$$N_1(t) = \frac{1}{2}N_{total}e^{-A_{10}t}$$

D) (5 points) Solve for $N_2(t)$ with the same initial condition as B.

$$N_2(t) = 0$$