HW7 problem 1

$$N_k dk = ?$$

Volume of circular shell

$=2\pi k dk/4$

4 is for upper right quadrant

Number of states in area=
area x States/area

States/area = $1 / (\pi/L)^2$:

$$N_k dk = \left(2\pi k dk / 4\right) \cdot \left(\frac{1}{(\pi / L)^2} \right) \cdot 2 = L^2 \frac{kdk}{2\pi}$$

$$\rho_k dk \equiv \frac{N_k dk}{\text{volume}} = \frac{kdk}{2\pi}$$

$$\rho(E)dE = ?$$

We use:

$$\rho_k dk = \rho(E)dE$$

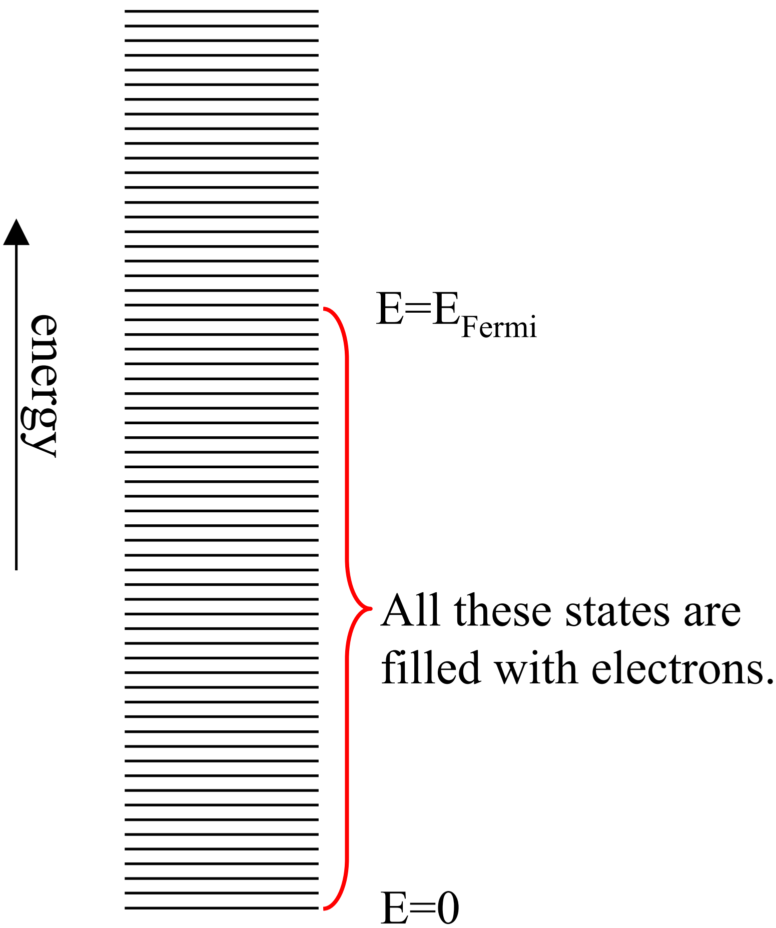
$$\rho_k dk = \frac{kdk}{2\pi}$$

$$E = \frac{\hbar^2 k^2}{2m} \Rightarrow k = \sqrt{\frac{2mE}{\hbar^2}} \Rightarrow dk = \sqrt{\frac{2m}{\hbar^2}} \frac{dE}{2\sqrt{E}}$$

$$\boxed{\rho(E)dE = \frac{m}{\pi\hbar^2} dE}$$

Fermi energy:

$$\#\,\text{electrons} = \int_0^{E_f} N_E \, dE = \int_0^{E_f} L^2 \frac{m}{\pi\hbar^2} dE = E_f L^2 \frac{m}{\pi\hbar^2}$$



$$\Rightarrow E_f = \frac{\hbar^2 \pi}{m}\left(\frac{\#\,\text{electrons}}{L^2}\right)$$

HW7 problem 2

$$\Psi(\vec{r},t) = \sum_n A_n e^{i(k_n x - \omega_n t)}$$

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}\right)\Psi(\vec{r},t) = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}\right)\left(\sum_n A_n e^{i(k_n x - \omega_n t)}\right)$$

$$= -\sum A_n \frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}\right)\left(e^{i(k_n x - \omega_n t)}\right) = -\sum A_n \frac{\hbar^2 k_n{}^2}{2m}\left(e^{i(k_n x - \omega_n t)}\right)$$

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\sum_n A_n e^{i(k_n x - \omega_n t)} = \sum_n A_n i\hbar\frac{\partial}{\partial t} e^{i(k_n x - \omega_n t)}$$

$$= \sum_n A_n(-i\omega_n)e^{i(k_n x - \omega_n t)} = \sum_n A_n\left(-\frac{\hbar^2 k_n{}^2}{2m}\right)e^{i(k_n x - \omega_n t)} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}\right)\Psi(\vec{r},t)$$

HW7 problem 3

$$n_i = \int_{E_c}^{\infty} P(E)\, \rho(E)\, dE$$

$$\rho(E) \cdot dE = \frac{2^{\frac{1}{2}} m^{\frac{3}{2}}}{\pi^2 \hbar^3} (E - E_c)^{\frac{1}{2}}\, dE$$

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1} \quad \text{when } E - E_F \gg kT \quad \approx e^{-(E-E_F)/kT}$$

$$n_i = \int_{E_c}^{\infty} \frac{2^{\frac{1}{2}} m^{\frac{3}{2}}}{\pi^2 \hbar^3} (E - E_c)^{\frac{1}{2}} \frac{1}{e^{(E-E_F)/kT}}\, dE$$

$$= \frac{2^{\frac{1}{2}} m^{\frac{3}{2}}}{\pi^2 \hbar^3} \int_{E_c}^{\infty} \frac{(E - E_c)^{\frac{1}{2}}}{e^{(E-E_F)/kT}}\, dE$$

Let $x = \frac{E - E_c}{kT}$, $x_c = \frac{E_F - E_c}{kT}$

$$dx = \frac{dE}{kT} \qquad x - x_c = \frac{E - E_f}{kT}$$

$$x(E_c) = 0$$

$$\int_{E_c}^{\infty} \frac{(E - E_c)^{\frac{1}{2}}}{e^{(E-E_F)/kT}}\, dE \Longrightarrow \int_0^{\infty} \frac{(kT)^{\frac{3}{2}} x^{\frac{1}{2}}}{e^{(x - x_c)}}\, dx$$

from tables or mathematica $= (kT)^{\frac{3}{2}} \frac{\sqrt{\pi}}{2} e^{x_c} = (kT)^{\frac{3}{2}} \frac{\sqrt{\pi}}{2} e^{(E_F - E_c)/kT}$

$$n_i = \frac{2^{\frac{1}{2}} m^{\frac{3}{2}}}{\pi^2 \hbar^3} (kT)^{\frac{3}{2}} \frac{\sqrt{\pi}}{2} e^{(E_F - E_c)/kT}$$

$$= 2 \frac{(kT)^{\frac{3}{2}} m^{\frac{3}{2}}}{\pi^{\frac{3}{2}} \hbar^{3 \cdot \frac{2}{2}} 2^{\frac{3}{2}}} e^{(E_F - E_c)/kT} = 2\left(\frac{kTm}{\pi \hbar^2 2}\right)^{\frac{3}{2}} e^{(E_F - E_c)/kT}$$

$$n_i = 2\left(\frac{m^* kT}{2\pi \hbar^2}\right)^{\frac{3}{2}} e^{(E_F - E_c)/kT}$$

HW7 problem 4

4. $p - n + N_D - N_A = 0$

$pn = n_i^2$

$n = N_c e^{(E_F - E_c)/kT}$

$n_i = N_c e^{(E_i - E_c)/kT}$

$N_c = n_i e^{(E_c - E_i)/kT}$

$n = n_i e^{(E_F - E_i)/kT}$

For n type N_d

$n \approx N_d = n_i e^{(E_F - E_i)/kT}$

$E_F - E_i = kT \ln \frac{N_D}{n_i}$

$p = N_v e^{(E_v - E_c)/kT}$

$n_i = N_v e^{(E_v - E_i)/kT}$

$N_v = n_i e^{(E_i - E_v)/kT}$

$p = n_i e^{(E_i - E_F)/kT}$

For p-type N_a

$p \approx N_a = n_i e^{(E_i - E_F)/kT}$

$E_i - E_F = kT \ln \frac{N_A}{n_i}$

HW7 problem 5

$$f_n(E_2) > f_v(E_1)$$

$$\frac{1}{e^{(E_2-F_n)/kT}+1} > \frac{1}{e^{(E_1-F_p)/kT}+1}$$

$$e^{(E_1-F_p)/kT}+1 > e^{(E_2-F_n)/kT}+1$$

$$E_1 - F_p > E_2 - F_n$$

$$F_n - F_p > E_2 - E_1$$

$$E_2 - E_1 > h\upsilon$$

$$F_n - F_p > h\upsilon$$