

NAME SOLUTION SET AND GRADING CRITERIA
ID _____

EECS 275B

Final exam

Open notes, open book (no wireless communications, though)

1:30-3:30 PM

1	2A	2B	2C	2D	2E	2	Total
/10	/15	/15	/15	/15	/15	/15	/100

- 1) (20 points) A new compound is discovered by your professor, “Burkonium”. The gain of Burkonium is *homogenously* broadened so that the gain bandwidth is 10^{12} Hz. It is put inside a cavity of length 1 meter. How many modes will oscillate? (Be careful.)

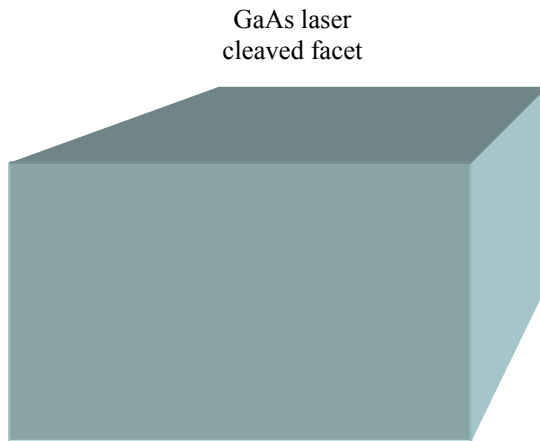
The correct answer is that only one mode will oscillate.

See page 211 of the book:

The fact that the laser oscillates on only one cavity mode is a consequence of the assumption of homogenous broadening.

1 point partial credit is given if the number of modes is calculated. (approximately 6600).

- 2) Consider an edge-emitting (cleaved) optically pumped GaAs laser. The length is 1 cm. The mirror is the cleaved facet, $n=3.5$. The quasi-Fermi level for the electrons is 0.1 eV above the conduction band edge. The quasi-Fermi level for the holes is 0.1 eV below the valence band edge.



A) At what wavelength is the gain maximized? (We will call this λ_{\max} .)

The gain is maximized at $hf = hc/\lambda = F_n - F_p = 0.1 \text{ eV} + 0.1 \text{ eV} + 1.43 \text{ eV} = 1.63 \text{ eV}$

Hence $\lambda_{\max} = hc/1.63 \text{ eV} = 760 \text{ nm}$.

Full credit if answer within 10% of correct answer.

10 points partial credit if technique is correct but numerical answer off by more than 10%.

B) If the gain coefficient at λ_{\max} is 2 cm^{-1} , and the length of the piece of GaAs is 1 cm , sketch the gain vs. frequency. (Use linear units for the gain). No units, no credit.

From the notes (lecture 13, slide 17), the frequency dependence of the gain is:

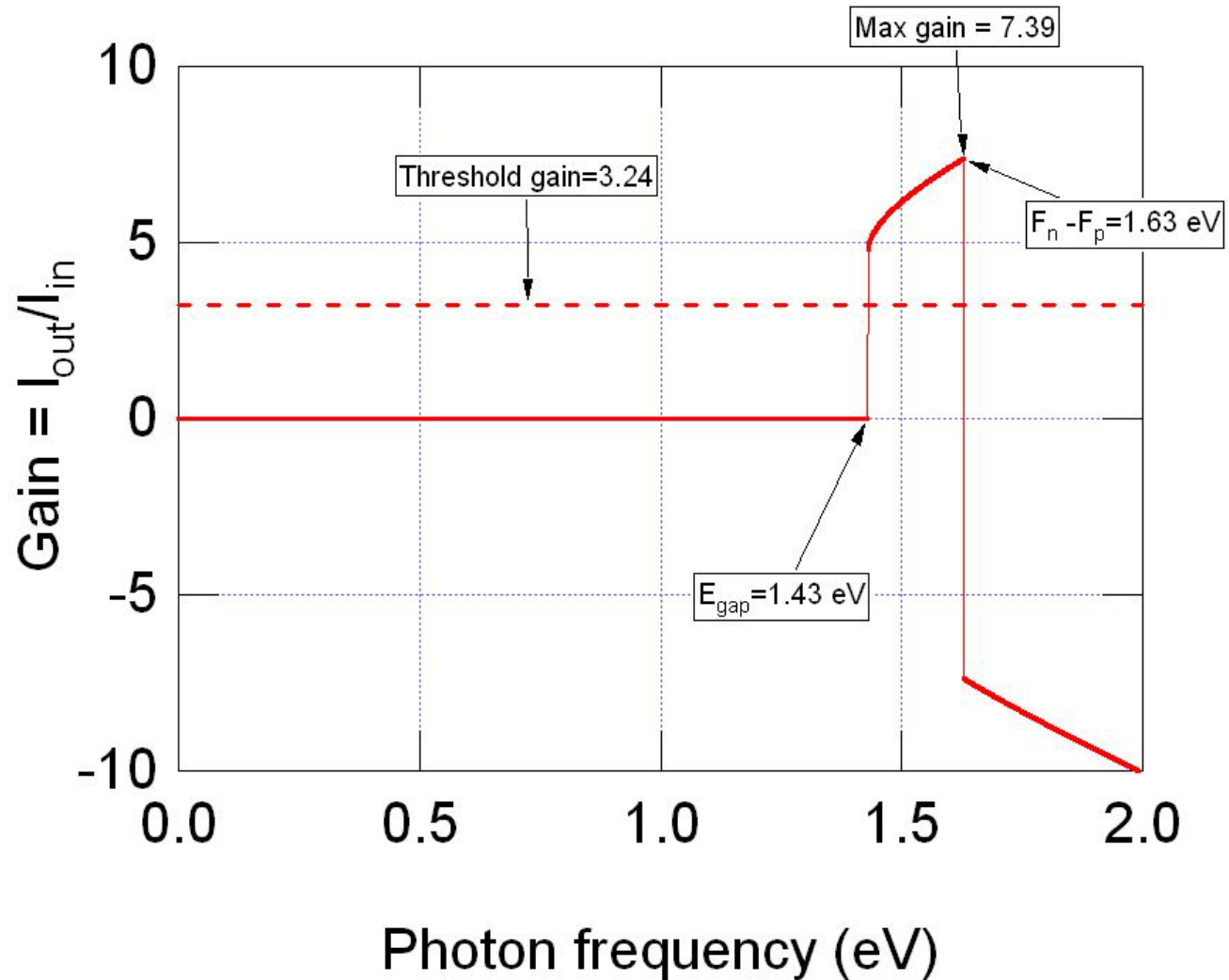
$$\gamma(\nu) \approx K \cdot \sqrt{h\nu - E_{\text{gap}}}$$

So the gain vs. frequency looks like:

$$G = e^{(\text{constant}) \cdot \sqrt{h\nu - E_{\text{gap}}}}$$

We know the maximum gain coefficient is 2 cm^{-1} , so the gain at maximum must be:

$$G = e^{\gamma L} = e^{(2 \text{ cm}^{-1})(1 \text{ cm})} = e^2 = 7.39$$



3 points if you plotted γ vs f correctly but magnitude of γ incorrect, but not the gain.

5 points if you plotted γ vs f correctly including magnitude of γ , but not the gain.

7 points for drawing the frequency dependence of the gain correctly.

15 points if you got the max gain correct, and the frequency dependence of the gain.

C) On the same graph, indicate what the minimum threshold gain is for lasing.

The optical loss at the boundary is $((n-1)/(n+1))^2 = 0.308$. So the minimum gain to exceed this loss is $1/0.308 = 3.24$.

D) Calculate the free spectral range. Be careful, because the free spectral range contains the index of refraction in it. (In class we neglected this, but on the final I expect you to take it into account).

From the book page 147, the free spectral range is $c/(2 n d)$ where d is the length (1 cm), and n the index of refraction. Thus, $FSR = 4.27 \text{ GHz} = 26.8 \cdot 10^9 \text{ rad/s}$.

Note about units: Hz is 1/second, and is used for f .
 ω is radians/second = $2 \pi f$.

If you expressed ω in Hz, you used the incorrect units and only 10 pts. are given.

5 points if you confused index of refraction and mode index. (Both use symbol n .)

E) Roughly how many modes will lase?

Only modes with gain above the threshold gain in the following graph will lase.

By eye, it certainly cannot be more than $(1.63 \text{ eV} - 1.43 \text{ eV})/h / 4.27 \text{ GHz} = 11241$.

1 point if you understood that number of modes = gain bandwidth/FSR.

14 points if you used correct approach but had wrong answers to B,D resulting in wrong E answer.

F) If there was some loss within the cavity due to optical absorption, say, everything else stayed the same, what is the maximum amount of loss that could be tolerated before lasing would not be possible? Express your answer to this part in decibels.

The max gain is 7.39. Therefore the total loss can be $1/7.39=0.135$. But, there is already loss due to the finite reflectivity of the cleaved mirrors of 0.308. So for a total loss of 0.135, we can tolerate an excess loss of 0.438. (Then, the total loss would be $0.438 \times 0.308 = 0.135$). So, the extra loss we can tolerate and still achieve lasing in decibels is $10 \log (0.438) = -3.6 \text{ dB}$.